M125
Fall 2003

Quiz 5 SOLUTIONS

Show all your work!

Suppose you have a wire that is 10 meters long. You cut the wire, and with one piece bend it into the shape of a square. With the other piece, bend it into the shape of a circle.

How should the wire be cut in order to have a minimum enclosed area?

Let x be the length of the wire cut for the square, so that $10 - x$ is the length for the circle. Since the circumference of a circle is $2\pi r$, the radius of our circle will be $r = \dfrac{10-x}{2\pi}$.

The full area is then:

$$A(x) = \frac{1}{16}x^2 + \pi\left(\frac{10-x}{2\pi}\right)^2 = \frac{1}{16}x^2 + \frac{1}{4\pi}(10-x)^2$$

where $0 \le x \le 10$.

If $x = 0$, all the wire goes for the circle, and the area is $\frac{25}{\pi} \approx 7.96$.

If $x = 10$, all the wire goes for the square, and the area is $\frac{1}{16} \cdot 100 \approx 6.25$.

Otherwise, we look for the critical points:

$$A'(x) = \frac{1}{8}x + \frac{1}{2\pi}(10-x)(-1) = 0$$

so that the derivative is zero if $x = \dfrac{40}{\pi+4} \approx 5.601$.

Put this back into the area, and we get:

$$A\left(\frac{40}{\pi+4}\right) = \frac{25}{\pi+4} \approx 3.5006$$

Looking back at the endpoints, this is the minimum value, obtained when $x \approx 5.6$ and $10 - x \approx 4.4$