M125
Fall 2003

Quiz 6

Name:__________

This ended up being a take home quiz:

If 1200 cm^2 is available to make a box with a square base and open top, find the dimensions that give the box with maximum volume.

SOLUTION: Let x, x, h be the dimensions of the box. Then the surface area of the box is given by the area of the base plus the area of the four sides. This quantity must be 1200:

$$1200 = x^2 + 4xh$$

Now find the maximum volume: $V = x^2 h$

From the surface area, we can solve for h, then substitute that into the formula for volume:

$$h = \frac{1200 - x^2}{4x} = \frac{300}{x} - \frac{1}{4}x$$

Before we continue, notice that there is a restriction on how large x can be and still have positive height- $x \leq \sqrt{1200}$. Furthermore, $x \geq 0$. At these extremes, the volume is 0.

Now make volume depend only on x, and find the critical points:

$$V = x^2 \left(\frac{300}{x} - \frac{1}{4}x \right) = 300x - \frac{1}{4}x^3$$

Take the derivative and set it equal to zero:

$$\frac{dV}{dx} = 300 - \frac{3}{4}x^2 = 0 \quad \Rightarrow \quad \frac{3}{4}x^2 = 300 \quad \Rightarrow \quad x = \sqrt{400} = 20$$

With this value of x, the dimensions of the box are $20 \times 20 \times 10$, and the maximum volume is 4000.