M125
Fall 2003

Quiz 7

SOLUTIONS

1. Differentiate (you do not need to simplify):

   (a) $y = x^3 + 3^x$
   $$y' = 3x^2 + 3^x \ln(3)$$
   General rule to use: $\frac{d}{dx}a^x = a^x \ln(a)$

   (b) $y = x\ln(x) + \tan^{-1}(x)$
   $$y' = \ln(x) + x \cdot \frac{1}{x} + \frac{1}{1+x^2}$$
   That is, use the product rule on the first term.

   (c) $y = \mathrm{e}^{\cos(2x)}$
   $$y' = -2\sin(2x)\mathrm{e}^{\cos(2x)}$$
   General rule: $\frac{d}{dx}\mathrm{e}^{f(x)} = f'(x)\mathrm{e}^{f(x)}$

2. Solve for x:

   (a) $\mathrm{e}^x - 2\mathrm{e}^{-2x} = 0$
   Many different possibilities here. We can write:
   $$\mathrm{e}^x = 2\mathrm{e}^{-2x} \quad \Rightarrow \quad \frac{\mathrm{e}^x}{\mathrm{e}^{-2x}} = 2 \quad \Rightarrow \quad \mathrm{e}^{3x} = 2 \quad \Rightarrow \quad 3x = \ln(2)$$
   so that $x = \dfrac{\ln(2)}{3}$

   (b) $\ln(x-2) - \ln(5) = 1$
   Rewrite using the rules of logarithms:
   $$\ln(x-2) - \ln(5) = 1 \quad \Rightarrow \quad \ln\left(\frac{x-2}{5}\right) = 1 \quad \Rightarrow \quad \frac{x-2}{5} = \mathrm{e} \quad \Rightarrow x = 5\mathrm{e} + 2$$

   (c) $\dfrac{\mathrm{e}^x}{1+\mathrm{e}^x} = \dfrac{1}{4}$
   Clear fractions (or cross multiply):
   $$4\mathrm{e}^x = 1 + \mathrm{e}^x \quad \Rightarrow \quad 3\mathrm{e}^x = 1 \quad \Rightarrow \quad \mathrm{e}^x = \frac{1}{3} \quad \Rightarrow x = -\ln(3)$$