Summary: Exponentials and Logs

(1) The rules of exponents and logs, and some definitions.

Exponentials	Logarithms
$y = a^x, a > 0$	$y = \log_a(x)$
Domain: All Reals	Domain: $x > 0$
Range: $y > 0$	Range: All Reals
Points on the Graph:	
$(0, 1)$	$(1, 0)$
$(1, a)$	$(a, 1)$
$(-1, 1/a)$	$(1/a, -1)$
Rules for manipulation:	
NOTE: $a^b = c$ is equivalent to $\log_a(c) = b$	
$a^0 = 1$	$\log_a(1) = 0$
$a^{\log_a(x)} = x, x > 0$	$\log_a(a^x) = x$, all x
$a^x a^y = a^{x+y}$	$\log_a(xy) = \log_a(x) + \log_a(y)$
$1/(a^x) = a^{-x}$	$\log_a(1/x) = -\log_a(x)$
	$\log_a(x/y) = \log_a(x) - \log_a(y)$
$a^{p/q} = \sqrt[q]{a^p}$	
$(a^x)^y = a^{xy}$	
$(ab)^x = a^x b^x$	
	$\log_a(x^r) = r\log_a(x)$
	$\log_a(x) = \frac{\log_c(x)}{\log_c(a)}$

(2) Base e:

(a) Define e to be that number that is approached by taking n sufficiently large: $\left(1 + \frac{1}{n}\right)^n$. Thus, e appears as a *natural growth* constant.
Numerically, e is approximately 2.7182818284...
e is irrational, so its decimal expansion is non-repeating.
Treat e as you would π.

(b) Definition: The natural logarithm: $\log_e(x) = \ln(x)$.

(c) $y = e^x$ and $y = \ln(x)$ are inverses of each other, so

$$e^{\ln(x)} = x \text{ for } x > 0, \text{ and } \ln(e^x) = x \text{ for all } x$$

(d) All the rules above apply; replace a by e and $\log_e$ by ln.

(e) All scientific calculators can compute using base e, so we can use it to compute other logs. For example,

$$\log_3(5) = \frac{\ln(5)}{\ln(3)}$$