Worked Problems: Exponentials and Logs

The following problems have detailed solutions on the internet. Work through them for practice using the rules of exponents and logs!

1. Rewrite the following to its equivalent form. That is, given an exponential expression, rewrite to a logarithmic expression; and given a logarithmic expression, rewrite to an exponential form.

   WORKED EXAMPLE: $\log_3(x-5) = 4 \;\Rightarrow\; 3^4 = x-5$

   WORKED EXAMPLE: $3^{x-5} = 2 \;\Rightarrow\; \log_3(2) = x-5$

   (a) $\log_2(x) = 3$

   (b) $\mathrm{e}^x\mathrm{e}^5 = 4$

   (c) $2\ln(x) + \ln(x-1) = 4$

   (d) $3^{x-5}2^x = 5$ HINT- You might need to use the following: Any number $a > 0$ can be written as $\mathrm{e}^{\ln(a)}$ and $a^b a^c = a^{b+c}$

   (e) $\ln(3x) - 4\ln(x+2) = 6$

2. One use of logarithms is to "get the exponent down front". For example, to solve $3^x = 5$, we can take the log (base anything) of both sides:

$$\log(3^x) = \log(5) \Rightarrow x\log(3) = \log(5) \Rightarrow x = \frac{\log(5)}{\log(3)}$$

   Solve the following for x:

   (a) $5^{2x-3} = 4$

   (b) $3^{x(x-1)} = 2$

   (c) $\log_2(x+3) - \log_2(x) = 1$

   (d) $2\log_9\left(\frac{x}{3}\right) = 1$

   (e) $\log_4\left(\frac{1}{2x}\right) = 3$

   (f) $12^{1/(x-1)} = 4$

Miscellaneous questions:

1. Determine the domain of $f(x) = \ln(x(x-1))$. For which x can this be rewritten as $\ln(x) + \ln(x-1)$?

2. If $f(x) = 2^x$ and $g(x) = log_2(x)$, compute $f(g(x))$ and $g(f(x))$, and find the domain of each.

3. If $f(x) = 3^x$, what is $f^{-1}(x)$?

4. Rewrite 4 as e^A for an appropriate A.