Extra Practice SOLUTIONS: Function Notation and Sign Charts

1. Let $f(x) = \sqrt{x}$. Find an expression for $f(ax+b)$. Find an expression for $f(x+h)-f(x)$.

   If $f(x) = \sqrt{x}$, then $f(ax+b) = \sqrt{ax+b}$. Also, $f(x+h) = \sqrt{x+h}$, so that

   $$f(x+h) - f(x) = \sqrt{x+h} - \sqrt{x}$$

2. Let $f(x) = \frac{2}{x}$. Find and simplify the expression for $\dfrac{f(x+h)-f(x)}{h}$

   In this case, $f(x+h) = \frac{2}{x+h}$, and

   $$\frac{f(x+h)-f(x)}{h} = \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \frac{\frac{x}{x} \cdot \frac{2}{x+h} - \frac{x+h}{x+h} \cdot \frac{2}{x}}{\frac{h}{1}} = \frac{2x - 2(x+h)}{x(x+h)} \cdot \frac{1}{h} =$$

   $$\frac{-2h}{x(x+h)} \cdot \frac{1}{h} = \frac{-2}{x(x+h)}$$

3. Let $f(x) = x^2 - 3x$. Find and simplify the expression for $f(3x-4)$. Find and simplify the expression for $f(x+h) - f(x)$.

   The expression for $f(3x-4)$ is $(3x-4)^2 - 3(3x-4)$. Simplify this to get $f(3x-4) = 9x^2 - 33x + 28$.

   Similarly, $f(x+h) = (x+h)^2 - 3(x+h) = x^2 + 2xh + h^2 - 3x - 3h$, so that

   $$f(x+h) - f(x) = 2xh + h^2 - 3h$$

4. Let $f(x) = 4$. Find and simplify the expression for $\dfrac{f(x+h)-f(x)}{h}$

   Since f of anything is 4,

   $$\frac{f(x+h)-f(x)}{h} = \frac{4-4}{h} = 0$$

5. Let $f(x) = 2^x$, $x = 1$ and $h = 2$. Find and simplify the expression for $\dfrac{f(x+h)-f(x)}{h}$.

   We might get the algebraic expression first, then plug in values:

   $$\frac{f(x+h)-f(x)}{h} = \frac{2^{x+h} - 2^x}{h} = \frac{2^{1+2} - 2^1}{2} = \frac{6}{2} = 3$$

NOTE: The next few exercises deal with using a sign chart. In general, use a sign chart to *determine where an expression is positive or negative.*

6. Find the domain: $f(x) = \sqrt{\dfrac{3x - x^2}{x+2}}$

   Use a sign chart- we want $\dfrac{x(3-x)}{x+2} \geq 0$

x	$-$	$-$	$+$	$+$
$3-x$	$+$	$+$	$+$	$-$
$x+2$	$-$	$+$	$+$	$+$
	$x < -2$	$-2 < x < 0$	$0 < x < 3$	$x > 3$
	POS	NEG	POS	NEG

   From this, we see that $x < -2$ or $0 < x < 3$. We can also include the points 0 and 3 but not $x = -2$, so our final answer is:

$$x < -2 \text{ or } 0 \leq x \leq 3$$

7. Find the domain: $f(x) = \ln\left(x(x+2)(x-3)\right)$

   Again, use a sign chart to determine where $x(x+2)(x-3) > 0$:

x	$-$	$-$	$+$	$+$
$x+2$	$-$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$
	$x < -2$	$-2 < x < 0$	$0 < x < 3$	$x > 3$
	NEG	POS	NEG	POS

   From this, we see that $-2 < x < 0$ or $x > 3$. We cannot include $x = 0, -2$ or 3, since $\ln(0)$ is not defined.

8. Solve for x, if $x^3 - x \geq 0$

   Sign chart again- Factor completely first: $x(x+1)(x-1) \geq 0$

x	$-$	$-$	$+$	$+$
$x+1$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$+$
	$x < -1$	$-1 < x < 0$	$0 < x < 1$	$x > 1$
	NEG	POS	NEG	POS

   Therefore, $x^3 - x \geq 0$ for $-1 \leq x \leq 0$ or $x \geq 1$

9. The following are False; explain why. If possible, change the statement so that it is true.

   (a) Let $a, b > 0$. Then $\sqrt{a^2 + b^2} = a + b$
   This is not a rule of algebra. We cannot simplify $\sqrt{a^2 + b^2}$.

(b) $\ln(x(x-1)/x+2) = \ln(x) + \ln(x-1) - \ln(x+2)$ for all x.
This is false because of **for all** x. It is true for all $x > 1$ (that is, where $x, x-1$ and $x+2$ are all positive).

(c) Let $f(x) = x^{-1}$. Then

$$\frac{f(x+h) - f(x)}{h} = \frac{x^{-1} + h - x^{-1}}{h} = 1$$

This is false because $f(x+h)$ has not been computed correctly. If $f(x) = x^{-1}$, then $f(x+h) = (x+h)^{-1} = \frac{1}{x+h}$. Now,

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \left(\frac{x - (x+h)}{x(x+h)}\right) \cdot \frac{1}{h} = \frac{-1}{x(x+h)}$$

(d) $\ln(a+b) = \ln(a) + \ln(b)$ if $a, b > 0$.
This is false because we cannot simplify $\ln(a+b)$.

(e) $(a+b)^2 = a^2 + b^2$.
This is false because we have forgotten the "middle terms":

$$(a+b)^2 = a^2 + 2ab + b^2$$

10. Misc. Algebra problems:

(a) Solve for y: $x = \dfrac{6y-5}{y+1}$

$$x(y+1) = 6y - 5 \Rightarrow xy + x = 6y - 5 \Rightarrow xy - 6y = -5 - x \Rightarrow$$
$$y(x-6) = -(x+5) \Rightarrow y = -\frac{x+5}{x-6}$$

(b) Solve for x in terms of y, z: $\dfrac{6}{x} = \dfrac{11}{y} + \dfrac{15}{z}$

$$\frac{6}{x} = \frac{11z + 15y}{yz} \Rightarrow 6yz = (11z + 15y)x \Rightarrow x = \frac{6yz}{11z + 15y}$$

NOTE: The following is a common **ERROR:** $\dfrac{x}{6} = \dfrac{y}{11} + \dfrac{z}{15} \Rightarrow x = \dfrac{6y}{11} + \dfrac{6z}{15}$

This is because $\dfrac{a}{b+c} \neq \dfrac{a}{b} + \dfrac{a}{c}$ (but we **can** write $\frac{a+b}{c}$ as $\frac{a}{c} + \frac{b}{c}$)

(c) Simplify and write without negative exponents: $\dfrac{4x^{-9}y^{-5}}{9x^{-3}y^{-9}} = \dfrac{4y^{9-5}}{9x^{9-3}} = \dfrac{4y^4}{9x^6}$

(d) Simplify: $\left(x^{4/7}y^{-4/9}\right)^{9/4} = x^{\frac{4}{7}\cdot\frac{9}{4}}y^{-\frac{4}{9}\cdot\frac{9}{4}} = \frac{x^{9/7}}{y}$
NOTE: Recall that $x^{p/q} = (\sqrt[q]{x})^p$

(e) Simplify: $\dfrac{\frac{3s^2-48}{s^2+2s-8}}{\frac{7s-28}{s^2-4s+4}} = \dfrac{3(s+4)(s-4)}{(s+4)(s-2)} \cdot \dfrac{(s-2)^2}{7(s-4)} = \dfrac{3}{7}(s-2)$