

## Calculus I Review Questions

**NOTE: You should also look over your old exams, old review sheets, old quizzes and homework.**

1. Compare and contrast the three “Value Theorems” of the course. When you would typically use each.
2. List the three things we need to check to see if a function  $f$  is continuous at  $x = a$ .
3. Derive the formula for the derivative of  $y = \sec^{-1}(x)$ .
4. Find the point on the line  $6x + y = 9$  that is closest to the point  $(-3, 1)$ .
5. Write the equation of the line tangent to  $x = \sin(2y)$  at  $x = 1$ .
6. For what values of  $A, B, C$  will  $y = Ax^2 + Bx + C$  satisfy the differential equation:

$$\frac{1}{2}y'' - 2y' + y = 3x^2 + 2x + 1$$

7. Compute the derivative of  $y$  with respect to  $x$ :

- (a)  $y = \sqrt[3]{2x+1}\sqrt{3x-2}$
- (b)  $y = \frac{1}{1+u^2}$ , where  $u = \frac{1}{1+x^2}$
- (c)  $\sqrt[3]{y} + \sqrt[3]{x} = 4xy$
- (d)  $\sqrt{x+y} = \sqrt[3]{x-y}$
- (e)  $y = \sin(2\cos(3x))$
- (f)  $y = (\cos(x))^{2x}$
- (g)  $y = (\tan^{-1}(x))^{-1}$
- (h)  $y = \sin^{-1}(\cos^{-1}(x))$
- (i)  $y = \log_{10}(x^2 - x)$
- (j)  $y = x^{x^2+2}$
- (k)  $y = e^{\cos(x)} + \sin(5x)$
- (l)  $y = \cot(3x^2 + 5)$
- (m)  $y = \sqrt{\sin(\sqrt{x})}$
- (n)  $\sqrt{x} + \sqrt[3]{y} = 1$
- (o)  $x \tan(y) = y - 1$
- (p)  $y = \frac{-2}{\sqrt[4]{t^3}}$ , where  $t = \ln(x^2)$ .
- (q)  $y = x3^{-1/x}$

8. Let  $f(x) = x2^{x+1}$ . Without explicitly computing the inverse, what is the equation of the tangent line to  $f^{-1}(x)$  at  $x = 4$ ? HINT: The point  $(1, 4)$  goes through the graph of  $f$ .

9. Find the local maximums and minimums:  $f(x) = x^3 - 3x + 1$  Show your answer is correct by using both the first derivative test and the second derivative test.

10. Compute the limit, if it exists. You may use any method (except a numerical table).

- (a)  $\lim_{x \rightarrow 0} \frac{x - \sin(x)}{x^3}$
- (b)  $\lim_{x \rightarrow 0} \frac{1 - e^{-2x}}{\sec(x)}$
- (c)  $\lim_{x \rightarrow 4^+} \frac{x - 4}{|x - 4|}$
- (d)  $\lim_{x \rightarrow -\infty} \sqrt{\frac{2x^2 - 1}{x + 8x^2}}$
- (e)  $\lim_{x \rightarrow \infty} \sqrt{x^2 + x + 1} - \sqrt{x^2 - x}$
- (f)  $\lim_{h \rightarrow 0} \frac{(1+h)^{-2} - 1}{h}$
- (g)  $\lim_{x \rightarrow \infty} x^3 e^{-x^2}$
- (h)  $\lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1}$
- (i)  $\lim_{x \rightarrow 0} \frac{x}{\tan^{-1}(4x)}$
- (j)  $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$

11. Determine all vertical/horizontal asymptotes and critical points of  $f(x) = \frac{2x^2}{x^2 - x - 2}$
12. Find values of  $m$  and  $b$  so that (1)  $f$  is continuous, and (2)  $f$  is differentiable.

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

13. Find the local and global extreme values of  $f(x) = \frac{x}{x^2 + x + 1}$  on the interval  $[-2, 0]$ .
14. Suppose  $f$  is differentiable so that:

$$f(1) = 1, f(2) = 2, f'(1) = 1, f'(2) = 2$$

If  $g(x) = f(x^3 + f(x^2))$ , evaluate  $g'(1)$ .

15. Let  $x^2y + a^2xy + \lambda y^2 = 0$

- (a) Let  $a$  and  $\lambda$  be constants, and let  $y$  be a function of  $x$ . Calculate  $\frac{dy}{dx}$ .
- (b) Let  $x$  and  $y$  be constants, and let  $a$  be a function of  $\lambda$ . Calculate  $\frac{da}{d\lambda}$ .

16. Show that  $x^4 + 4x + c = 0$  has at most one solution in the interval  $[-1, 1]$ .

17. True or False, and give a short explanation.

- (a) If  $f$  has an absolute minimum at  $c$ , then  $f'(c) = 0$ .  
 (b) If  $f$  is differentiable, then

$$\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$$

- (c)  $\frac{d}{dx}(10^x) = x10^{x-1}$   
 (d) If  $f'(x)$  exists and is nonzero for all  $x$ , then  $f(1) \neq f(0)$ .  
 (e) If  $y = ax + b$ , then  $\frac{dy}{da} = x$   
 (f) If  $2x + 1 \leq f(x) \leq x^2 + 2$  for all  $x$ , then  $\lim_{x \rightarrow 1} f(x) = 3$ .  
 (g) If  $f'(r)$  exists, then

$$\lim_{x \rightarrow r} f(x) = f(r)$$

- (h) If  $f$  and  $g$  are differentiable, then:

$$\frac{d}{dx}(f(g(x))) = f'(x)g'(x)$$

- (i) If  $f(x) = x^2$ , then the equation of the tangent line at  $x = 3$  is:  $y - 9 = 2x(x - 3)$   
 (j)  $\lim_{\theta \rightarrow \frac{\pi}{3}} \frac{\cos(\theta) - \frac{1}{2}}{\theta - \frac{\pi}{3}} = -\sin\left(\frac{\pi}{3}\right)$   
 (k) There is no solution to  $e^x = 0$   
 (l)  $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$   
 (m)  $5^{\log_5(2x)} = 2x$ , for  $x > 0$ .  
 (n)  $\frac{d}{dx} \ln(|x|) = \frac{1}{x}$ , for all  $x \neq 0$ .  
 (o) If  $x > 0$ , then  $(\ln(x))^6 = 6 \ln(x)$   
 (p) The most general antiderivative of  $x^{-2}$  is  $\frac{-1}{x} + C$ .

18. Find the domain of  $\ln(x - x^2)$ :

19. Find the value of  $c$  guaranteed by the Mean Value Theorem, if  $f(x) = \frac{x}{x+2}$  on the interval  $[1, 4]$ .

20. Given that the graph of  $f$  passes through the point  $(1, 6)$  and the slope of the tangent line at  $(x, f(x))$  is  $2x + 1$ , find  $f(2)$ .

21. A fly is crawling from left to right along the curve  $y = 8 - x^2$ , and a spider is sitting at  $(4, 0)$ . At what point along the curve does the spider first see the fly?

22. Compute the limit, without using L'Hospital's Rule.  $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$

23. For what value(s) of  $c$  does  $f(x) = cx^4 - 2x^2 + 1$  have both a local maximum and a local minimum?

24. If  $f(x) = \sqrt{1 - 2x}$ , determine  $f'(x)$  by using the definition of the derivative.

25. A *point of inflection* for a function  $f$  is the  $x$  value for which  $f''(x)$  changes sign (either from positive to negative or vice versa).

Find constants  $a$  and  $b$  so that  $(1, 6)$  is an inflection point for  $y = x^3 + ax^2 + bx + 1$ .

Hint: The IVT might come in handy

26. Suppose that  $F(x) = f(g(x))$  and  $g(3) = 6$ ,  $g'(3) = 4$ ,  $f(3) = 2$  and  $f'(6) = 7$ . Find  $F'(3)$ .

27. Find the dimensions of the rectangle of largest area that has its base on the  $x$ -axis and the other two vertices on the parabola  $y = 8 - x^2$ .

28. Let  $G(x) = h(\sqrt{x})$ . Then where is  $G$  differentiable? Find  $G'(x)$ .

29. If position is given by:  $f(t) = t^4 - 2t^3 + 2$ , find the times when the acceleration is zero. Then compute the velocity at these times.

30. If  $y = \sqrt{5t - 1}$ , compute  $y'''$ .

31. Find a second degree polynomial so that  $P(2) = 5$ ,  $P'(2) = 3$ , and  $P''(2) = 2$ .

32. Find a function  $f(x)$  so that  $f'(x) = 4 - 3(1 + x^2)^{-1}$ , and  $f(1) = 0$

33. If  $f(x) = (2 - 3x)^{-1/2}$ , find  $f(0)$ ,  $f'(0)$ ,  $f''(0)$ .

34. Car A is traveling west at 50 mi/h, and car B is traveling north at 60 mi/h. Both are headed for the intersection between the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

35. Compute  $\Delta y$  and  $dy$  for the value of  $x$  and  $\Delta x$ :  $f(x) = 6 - x^2$ ,  $x = -2$ ,  $\Delta x = 0.4$ .

36. Find the linearization of  $f(x) = \sqrt{1 - x}$  at  $x = 0$ .

37. Find  $f(x)$ , if  $f''(x) = t + \sqrt{t}$ , and  $f(1) = 1$ ,  $f'(1) = 2$ .

38. Find  $f'(x)$  directly from the definition of the derivative (using limits and without L'Hospital's rule):

(a)  $f(x) = \sqrt{3 - 5x}$

(b)  $f(x) = x^2$

- (c)  $f(x) = x^{-1}$
39. If  $f(0) = 0$ , and  $f'(0) = 2$ , find the derivative of  $f(f(f(f(x))))$  at  $x = 0$ .
40. Differentiate:
- $$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ -\sqrt{-x} & \text{if } x < 0 \end{cases}$$
- Is  $f$  differentiable at  $x = 0$ ? Explain.
41.  $f(x) = |\ln(x)|$ . Find  $f'(x)$ .
42.  $f(x) = xe^{g(\sqrt{x})}$ . Find  $f'(x)$ .
43. Find a formula for  $dy/dx$ :  $x^2 + xy + y^3 = 0$ .
44. Show that 5 is a critical number of  $g(x) = 2 + (x - 5)^3$ , but that  $g$  does not have a local extremum there.
45. Find the general antiderivative:
- $f(x) = 4 - x^2 + 3e^x$
  - $f(x) = \frac{3}{x^2} + \frac{2}{x} + 1$
  - $f(x) = \frac{1+x}{\sqrt{x}}$
46. Find the slope of the tangent line to the following at the point (3,4):  $x^2 + \sqrt{y}x + y^2 = 31$
47. Find the critical values:  $f(x) = |x^2 - x|$
48. Does there exist a function  $f$  so that  $f(0) = -1$ ,  $f(2) = 4$ , and  $f'(x) \leq 2$  for all  $x$ ?
49. Find a function  $f$  so that  $f'(x) = x^3$  and  $x+y = 0$  is tangent to the graph of  $f$ .
50. Find  $dy$  if  $y = \sqrt{1-x}$  and evaluate  $dy$  if  $x = 0$  and  $dx = 0.02$ . Compare your answer to  $\Delta y$
51. Fill in the question marks: If  $f''$  is positive on an interval, then  $f'$  is ? and  $f$  is ?.
52. If  $f(x) = x - \cos(x)$ ,  $x$  is in  $[0, 2\pi]$ , then find the value(s) of  $x$  for which
- $f(x)$  is greatest and least.
  - $f(x)$  is increasing most rapidly.
  - The slopes of the lines tangent to the graph of  $f$  are increasing most rapidly.
53. Show there is *exactly* one solution to:  $\ln(x) = 3 - x$ .
54. Approximate the change in volume of a cone, if we assume the height to be constant and  $r$  changes from 2 to 2.1. ( $V = \frac{1}{3}\pi r^2 h$ )
55. Sketch the graph of a function that satisfies all of the given conditions:
- $$\begin{array}{llll} f(1) = 5 & f(4) = 2 & f'(1) = f'(4) = 0 \\ \lim_{x \rightarrow 2^+} f(x) = \infty, & \lim_{x \rightarrow 2^-} f(x) = 3 & f(2) = 4 \end{array}$$
56. If  $s^2t + t^3 = 1$ , find  $\frac{dt}{ds}$  and  $\frac{ds}{dt}$ .
57. Find the specific antiderivative:
- $f'(x) = 3\sqrt{x} - \frac{1}{\sqrt{x}}$ ,  $f(1) = 2$
  - $f''(x) = x^2 + 3\cos(x)$ ,  $f(0) = 2, f'(0) = 3$
  - $f''(x) = 3e^x + 5\sin(x)$ ,  $f(0) = 1, f'(0) = 2$
  - $f'(x) = \frac{4}{\sqrt{1-x^2}}$ ,  $f(1/2) = 1$ .
58. Find the area of the largest rectangle that can be inscribed in a right triangle with legs measuring 3 cm and 4 cm, and two sides of the rectangle lie along the legs.
59. A piece of wire 10 m long is cut into two pieces—one piece is bent into the shape of a square, the other into an equilateral triangle. How should the wire be cut so that the total enclosed area is a minimum?
60. A kite 100 feet above the ground moves horizontally (it stays 100 feet above the ground) away from you at a rate of 8 ft/sec. At what rate is the angle between the string and the ground decreasing when 200 feet of string has been let out?