**Addition to Review Solutions, Exam 3, Calc I**

18. If $f(x) = 3x^5 - 5x^3 + 3$, find the intervals of increase or decrease, and the intervals where $f$ is concave up/down:

    To find where $f$ is increasing/decreasing, look for where $f'(x) > 0$ and $f'(x) < 0$ (sign chart):

$$f'(x) = 15x^4 - 15x^2 = 15x^2(x^2 - 1)$$

Sign Chart:

| | | | | |
|---|---|---|---|---|
| $x^2$ | $+$ | $+$ | $+$ | $+$ |
| $(x-1)$ | $-$ | $-$ | $-$ | $+$ |
| $(x+1)$ | $-$ | $+$ | $+$ | $+$ |
| | $x < -1$ | $-1 < x < 0$ | $0 < x < 1$ | $x > 1$ |

So $f$ is decreasing if $-1 < x < 0$ and $0 < x < 1$. $f$ is increasing if $x < -1$ or if $x > 1$.

For concave up/down, look at the sign of the second derivative:

$$f''(x) = 60x^3 - 30x = 30x(2x^2 - 1)$$

Sign Chart:

| | | | | |
|---|---|---|---|---|
| $30x$ | $-$ | $-$ | $+$ | $+$ |
| $2x^2 - 1$ | $+$ | $-$ | $-$ | $+$ |
| | $x < -1/\sqrt{2}$ | $-1/sqrt2 < x < 0$ | $0 < x < 1/sqrt2$ | $x > 1/sqrt2$ |

So $f$ is concave down if $x < -1/sqrt2$ or $0 < x < 1/sqrt2$. $f$ is concave up if $-1/sqrt2 < x < 0$ or $x > 1/sqrt2$.

19. Show that $x^4 + 4x + c = 0$ has at most 2 real solutions.

    Use Rolle's Theorem- If we can show that the derivative will be zero *at most once*, then $x^4 + 4x + c = 0$ at most twice.

    In this case,

$$4x^3 + 4 = 0 \quad \Rightarrow x^3 = -1 \quad \Rightarrow x = -1$$

So the derivative is zero only once. Therefore, there can be at most 2 solutions to $x^4 + 4x + c = 0$ (if there were three or more solutions, then the derivative would have to be zero more than once).

20. If $3 \le f'(x) \le 5$, for all $x$, show that $f(8) - f(2)$ must be between 18 and 30:

    By the Mean Value Theorem, there must be a $c$ in $(2, 8)$ such that:

$$\frac{f(8) - f(2)}{8 - 2} = f'(c) \quad \Rightarrow \quad \frac{f(8) - f(2)}{6} = f'(c)$$

We know that $f'(c)$ is between 3 and 5:

$$3 \le \frac{f(8) - f(2)}{6} \le 5 \quad \Rightarrow \quad 18 \le f(8) - f(2) \le 30$$