Summary: To Exam 1 (Up to 2.8)

General Background

These are just the things that we emphasized in our review- it's not an exhaustive list.

1. Specific skills we talked about: Construct the equation of a line, Use the quadratic formula, find trigonometric values from right triangles/unit circle, function composition, simplify exponentials/logs using rules of exponents/logs.
2. Definitions: $|x|$, "one-to-one", "natural domain"
3. Be able to "Find the domain".
4. Use a **sign chart** to determine where an expression is positive/negative.
5. Know the difference between "inverse of a function" and the reciprocal of a function.
6. Given a formula for $f(x)$, be able to compute expressions like $f(2+h)$.

The Limit

1. Definitions:

$$\lim_{x\to a^+} f(x) = L, \qquad \lim_{x\to a^-} f(x) = L$$

   - Word definition: $\lim_{x\to a} f(x) = L$ means that we can keep the $f(x)$ values arbitrarily close to L by keeping the $x-$values sufficiently close to a.
   - ϵ, δ definition: $\lim_{x\to a} f(x) = L$ means that, for every $\epsilon > 0$, there isa $\delta > 0$ such that

$$|f(x) - L| < \epsilon \text{ whenever } 0 < |x - a| < \delta$$

2. Algebraic Methods:
   (a) Simplify (e.g., absolute values)
   (b) Factor and Cancel
   (c) Multiply by Conjugate
   (d) Divide by x^n (Mainly for $x \to \infty$)
   Be careful! $x = \sqrt{x^2}$ if $x \geq 0$, but if $x < 0$, $x = -\sqrt{x^2}$
3. The Squeeze Theorem.
4. Horizontal/Vertical Asymptotes:
   (a) $x = a$ is a vertical asymptote for $f(x)$ if one of the following limits is infinite: $\lim_{x\to a^\pm} f(x)$
   (b) $y = b$ is a horizontal asymptote for $f(x)$ if one of the following is true: $\lim_{x\to\pm\infty} f(x) = b$ Our template function:

$$\lim_{x\to\pm\infty} \frac{1}{x^r} = 0, \quad r > 0$$

   (Note: x^r needs to be computable if $x \to -\infty$) Also, in a similar vein:

$$\lim_{x\to\infty} \mathrm{e}^{-x} = 0$$

   The inverse tangent has horizontal asymptotes:

$$\lim_{x\to\pm\infty} \tan^{-1}(x) = \pm\frac{\pi}{2}$$

   And in general, if a function has a vertical asymptote at $x = a$, its inverse function will have a horizontal asymptote at $y = a$.

5. Intuition that can be used:
   (a) "$\infty + \infty = \infty$", but $\infty - \infty$ is not necessarily 0.
   (b) If the denominator goes to zero, but the numerator does not, the limit is $\pm\infty$.
   (c) If the denominator goes to $\pm\infty$, and the numerator does not, the overall limit goes to zero.
6. Limit Laws (Sect 1.3)

Continuity

1. Definition:
$$\lim_{x \to a} f(x) = f(a)$$
Means 3 things: (1) $f(a)$ exists, (2) The limit exists, and (3) Items 1 and 2 are the same number.
2. Theory about continuous functions: Know that our usual functions (see the list in Theorem 7) are all continuous *on their domain*. Know that the sum/difference, product/quotient of continuous functions is continuous (with a possibly restricted domain)
3. IVT: If f is continuous on $[a, b]$, and w is a number between $f(a)$ and $f(b)$, there is at least one c in $[a, b]$ so that $f(c) = w$.
   In practice, we usually use the IVT as:
   If f is continuous, and $f(x_1) > 0, f(x_2) < 0$, then there is a c between x_1 and x_2 where $f(c) = 0$ (f has at least one root in the interval between x_1 and x_2).

The Derivative

1. Know the definition of Average Velocity and the technique we use to get Instantaneous Velocity (aka Velocity)
2. Definition:
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$
3. Interpretations of the Derivative of f at $x = a$:
   (a) The velocity at $x = a$.
   (b) The slope of the tangent line at $(a, f(a))$.
   (c) The instantaneous rate of change of f at $x = a$.
4. Equation of the Tangent Line at $x = a$: This is the line going through $(a, f(a))$ with slope $f'(a)$. The best (and fastest) way to write the line:
$$y - f(a) = f'(a)(x - a)$$
5. Be able to compute the derivative using the definition.