**Review Questions, Exam 3**
**Math 125, Fall 2006**

Exam 3 will cover material from 3.10-4.3. You should also look over the homework and quizzes.

1. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child paying out cord when 150 feet of cord is out? (Assume that the cord forms a line- actually an unrealistic assumption).

2. Use differentials to approximate the increase in area of a soap bubble, when its radius increases from 3 inches to 3.025 inches ($A = 4\pi r^2$)

3. True or False, and give a short reason:

   (a) If air is being pumped into a spherical rubber balloon at a constant rate, then the radius will increase, but at a slower and slower rate.

   (b) If $f(x), g(x)$ are increasing on an interval $I$, so is $f(x)g(x)$.

   (c) If $y = x^5$, then $dy \geq 0$

   (d) If a car *averages* 60 miles per hour over an interval of time, then at some instant, the speedometer must have read exactly 60.

   (e) A global maximum is always a local maximum.

   (f) The linear function $f(x) = ax + b$, where $a, b$ are constant, and $a \neq 0$, has no minimum value on any open interval. (An interval is open if it does not include its endpoints).

   (g) Suppose $P$ and $Q$ are two points on the surface of the sea, with $Q$ lying generally to the east of $P$. It is possible to sail from $P$ to $Q$ (always sailing roughly east), without *ever* sailing in the exact direction from $P$ to $Q$.

   (h) If $f(x) = 0$ has three (distinct) real solutions, then $f'(x) = 0$ must have (at least) two solutions (Assume $f$ is differentiable). Furthermore, $f''(x) = 0$ must have at least one solution.

4. Show that, if $f(x)$ is increasing then $1/f(x)$ is decreasing.

5. Explain the first and second derivative test. What are they testing for?

6. State the three "Value Theorems":

7. Compute $\Delta y$ and $dy$ for the given $x$ and $dx = \Delta x$. Sketch a diagram and label $\Delta x$, $\Delta y$ and $dy$, if $f(x) = 6 - x^2$, $x = -2$, $\Delta x = 1$.

8. Linearize at $x = 0$:
$$y = \sqrt{x} e^{x^2} (x^2 + 1)^{10}$$

9. Estimate by linear approximation the change in the indicated quantity.

   (a) The volume, $V = s^3$ of a cube, if its side length $s$ is increased from 5 inches to 5.1 inches.

(b) The volume, $V = \frac{4}{3}\pi r^3$ of a sphere, if its radius changes from 2 to 2.1

(c) The volume, $V = \frac{1000}{p}$, of a gas, if the pressure $p$ is decreased from 100 to 99.

(d) The period of oscillation, $T = 2\pi\sqrt{\frac{L}{32}}$, of a pendulum, if its length $L$ is increased from 2 to 2.2.

10. For the following problems, find where $f$ is increasing or decreasing. If asked, also check concavity.

(a) $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ (Also check for concave up/down)

(b) $f(x) = \frac{x}{x+1}$

(c) $f(x) = x\sqrt{x^2+1}$

11. Show that the given function satisfies the hypotheses of the Mean Value Theorem. Find all numbers $c$ in that interval that satisfy the conclusion of that theorem. For comparison purposes, given these functions and intervals, what would the Intermediate Value Theorem conclude? Finally, find the global max and global min for each function.

(a) $f(x) = x^3$, $[-1, 1]$

(b) $f(x) = \sqrt{x-1}$, $[2, 5]$

(c) $f(x) = x + \frac{1}{x}$, $[1, 5]$

12. Show that $f(x) = x^{2/3}$ does not satisfy the hypotheses of the mean value theorem on $[-1, 27]$, but nevertheless, there is a $c$ for which: $f'(c) = \dfrac{f(27) - f(-1)}{27 - (-1)}$ Find the value of $c$.

13. At 1:00 PM, a truck driver picked up a fare card at the entrance of a tollway. At 2:15 PM, the trucker pulled up to a toll booth 100 miles down the road. After computing the trucker's fare, the toll booth operator summoned a highway patrol officer who issued a speeding ticket to the trucker. (The speed limit on the tollway is 65 MPH).

(a) The trucker claimed that she hadn't been speeding. Is this possible? Explain.

(b) The fine for speeding is \$35.00 plus \$2.00 for each mph by which the speed limit is exceeded. What is the trucker's minimum fine?

14. Let $f(x) = \frac{1}{x}$

(a) What does the Extreme Value Theorem (EVT) say about $f$ on the interval $[0.1, 1]$?

(b) Although $f$ is continuous on $[1, \infty)$, it has no minimum value on this interval. Why doesn't this contradict the EVT?

15. Let $f$ be a function so that $f(0) = 0$ and $\frac{1}{2} \leq f'(x) \leq 1$ for all $x$. Use the Mean Value Theorem to explain why $f(2)$ cannot be 3.

16. Sketch the graph of a function that satisfies all of the given properties:

$$f'(-1) = 0, f'(1) \text{ does not exist}, f'(x) < 0 \text{ if } |x| < 1, f'(x) > 0 \text{ if } |x| > 1$$

$$f(-1) = 4, f(1) = 0, f''(x) > 0 \text{ if } x > 0$$

17. Find the local maximums and local minimums of $f$ using both the first and second derivative tests:

$$f(x) = x + \sqrt{1-x}$$

18. If $f(x) = 3x^5 - 5x^3 + 3$, find the intervals of increase or decrease, find the local max/min, find the intervals of concavity.

19. Show the equation $x^4 + 4x + c = 0$ has at most 2 real solutions.

20. If $3 \leq f'(x) \leq 5$ for all $x$, show that the change $f(8) - f(2)$ is at least 18 and at most 30.

21. Related Rates Extra Practice:

    (a) The top of a 25-foot ladder, leaning against a vertical wall, is slipping down the wall at a rate of 1 foot per second. How fast is the bottom of the ladder slipping along the ground when the bottom of the ladder is 7 feet away from the base of the wall?

    (b) A 5-foot girl is walking toward a 20-foot lamppost at a rate of 6 feet per second. How fast is the tip of her shadow (cast by the lamppost) moving?

    (c) Under the same conditions as above, how fast is the length of the girl's shadow changing?

    (d) A rocket is shot vertically upward with an initial velocity of 400 feet per second. Its height $s$ after $t$ seconds is $s = 400t - 16t^2$. How fast is the distance changing from the rocket to an observer on the ground 1800 feet away from the launch site, when the rocket is still rising and is 2400 feet above the ground?

    (e) A small funnel in the shape of a cone is being emptied of fluid at the rate of 12 cubic centimeters per second (the tip of the cone is downward). The height of the cone is 20 cm and the radius of the top is 4 cm. How fast is the fluid level dropping when the level stands 5 cm above the vertex of the cone [The volume of a cone is $V = \frac{1}{3}\pi r^2 h$].

    (f) A balloon is being inflated by a pump at the rate of 2 cubic inches per second. How fast is the diameter changing when the radius is $\frac{1}{2}$ inch?

    (g) A particle moves on the hyperbola $x^2 - 18y^2 = 9$ in such a way that its $y$ coordinate increases at a constant rate of 9 units per second. How fast is the $x-$coordinate changing when $x = 9$?

    (h) An object moves along the graph of $y = f(x)$. At a certain point, the slope of the curve is $\frac{1}{2}$ and the $x-$coordinate is decreasing at 3 units per second. At that point, how fast is the $y-$coordinate changing?

    (i) A rectangular trough is 8 feet long, 2 feet across the top, and 4 feet deep. If water flows in at a rate of 2 cubic feet per minute, how fast is the surface rising when the water is 1 foot deep?

    (j) If a mothball (sphere) evaporates at a rate proportional to its surface area $4\pi r^2$, show that its radius decreases at a constant rate.

    (k) If an object is moving along the curve $y = x^3$, at what point(s) is the $y-$coordinate changing 3 times more rapidly than the $x-$coordinate?