Calculus I Review Questions

NOTE: You should also look over your old exams, old review sheets, old quizzes and homework.

1. Compare and contrast the three "Value Theorems" of the course. When you would typically use each.

2. List the three things we need to check to see if a function f is continuous at $x = a$.

3. Derive the formula for the derivative of $y = \sec^{-1}(x)$.

4. Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$.

5. Write the equation of the line tangent to $x = \sin(2y)$ at $x = 1$.

6. For what values of A, B, C will $y = Ax^2 + Bx + C$ satisfy the differential equation:

$$\frac{1}{2}y'' - 2y' + y = 3x^2 + 2x + 1$$

7. Compute the derivative of y with respect to x:
   (a) $y = \sqrt[3]{2x+1}\sqrt[5]{3x-2}$
   (b) $y = \frac{1}{1+u^2}$, where $u = \frac{1}{1+x^2}$
   (c) $\sqrt[3]{y} + \sqrt[3]{x} = 4xy$
   (d) $\sqrt{x+y} = \sqrt[3]{x-y}$
   (e) $y = \sin(2\cos(3x))$
   (f) $y = (\cos(x))^{2x}$
   (g) $y = (\tan^{-1}(x))^{-1}$
   (h) $y = \sin^{-1}(\cos^{-1}(x))$
   (i) $y = \log_{10}(x^2 - x)$
   (j) $y = x^{x^2+2}$
   (k) $y = e^{\cos(x)} + \sin(5^x)$
   (l) $y = \cot(3x^2 + 5)$
   (m) $y = \sqrt{\sin(\sqrt{x})}$
   (n) $\sqrt{x} + \sqrt[3]{y} = 1$
   (o) $x\tan(y) = y - 1$
   (p) $y = \frac{-2}{\sqrt[4]{t^3}}$, where $t = \ln(x^2)$.
   (q) $y = x3^{-1/x}$

8. Let $f(x) = x2^{x+1}$. Without explicitly computing the inverse, what is the equation of the tangent line to $f^{-1}(x)$ at $x = 4$? HINT: The point $(1, 4)$ goes through the graph of f.

9. Find the local maximums and minimums: $f(x) = x^3 - 3x + 1$ Show your answer is correct by using both the first derivative test and the second derivative test.

10. Compute the limit, if it exists. You may use any method (except a numerical table).
   (a) $\lim_{x\to 0} \frac{x - \sin(x)}{x^3}$
   (b) $\lim_{x\to 0} \frac{1 - e^{-2x}}{\sec(x)}$
   (c) $\lim_{x\to 4^+} \frac{x-4}{|x-4|}$
   (d) $\lim_{x\to -\infty} \sqrt{\frac{2x^2-1}{x+8x^2}}$
   (e) $\lim_{x\to\infty} \sqrt{x^2+x+1} - \sqrt{x^2-x}$
   (f) $\lim_{h\to 0} \frac{(1+h)^{-2} - 1}{h}$
   (g) $\lim_{x\to\infty} x^3 e^{-x^2}$
   (h) $\lim_{x\to 1} \frac{x^{1000} - 1}{x - 1}$
   (i) $\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)}$
   (j) $\lim_{x\to 1} x^{\frac{1}{1-x}}$

11. Determine all vertical/horizontal asymptotes and critical points of $f(x) = \frac{2x^2}{x^2-x-2}$

12. Find values of m and b so that (1) f is continuous, and (2) f is differentiable.

$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

13. Find the local and global extreme values of $f(x) = \frac{x}{x^2+x+1}$ on the interval $[-2, 0]$.

14. Suppose f is differentiable so that:

$$f(1) = 1,\ f(2) = 2,\ f'(1) = 1\ f'(2) = 2$$

   If $g(x) = f(x^3 + f(x^2))$, evaluate $g'(1)$.

15. Let $x^2y + a^2xy + \lambda y^2 = 0$
   (a) Let a and λ be constants, and let y be a function of x. Calculate $\frac{dy}{dx}$:
   (b) Let x and y be constants, and let a be a function of λ. Calculate $\frac{da}{d\lambda}$:

16. Show that $x^4 + 4x + c = 0$ has at most one solution in the interval $[-1, 1]$.

17. True or False, and give a short explanation.

   (a) If f has an absolute minimum at c, then $f'(c) = 0$.

   (b) If f is differentiable, then

   $$\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$$

   (c) $\frac{d}{dx}(10^x) = x10^{x-1}$

   (d) If $f'(x)$ exists and is nonzero for all x, then $f(1) \neq f(0)$.

   (e) If $y = ax + b$, then $\frac{dy}{da} = x$

   (f) If $2x + 1 \leq f(x) \leq x^2 + 2$ for all x, then $\lim_{x\to 1} f(x) = 3$.

   (g) If $f'(r)$ exists, then

   $$\lim_{x\to r} f(x) = f(r)$$

   (h) If f and g are differentiable, then:

   $$\frac{d}{\mathrm{dx}}(f(g(x)) = f'(x)g'(x)$$

   (i) If $f(x) = x^2$, then the equation of the tangent line at $x = 3$ is: $y - 9 = 2x(x - 3)$

   (j) $\lim\limits_{\theta\to\frac{\pi}{3}} \dfrac{\cos(\theta) - \frac{1}{2}}{\theta - \frac{\pi}{3}} = -\sin\left(\dfrac{\pi}{3}\right)$

   (k) There is no solution to $\mathrm{e}^x = 0$

   (l) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

   (m) $5^{\log_5(2x)} = 2x$, for $x > 0$.

   (n) $\frac{d}{dx}\ln(|x|) = \frac{1}{x}$, for all $x \neq 0$.

   (o) If $x > 0$, then $(\ln(x))^6 = 6\ln(x)$

   (p) The most general antiderivative of x^{-2} is $\frac{-1}{x} + C$.

18. Find the domain of $\ln(x - x^2)$:

19. Find the value of c guaranteed by the Mean Value Theorem, if $f(x) = \frac{x}{x+2}$ on the interval $[1, 4]$.

20. Given that the graph of f passes through the point $(1, 6)$ and the slope of the tangent line at $(x, f(x))$ is $2x + 1$, find $f(2)$.

21. A fly is crawling from left to right along the curve $y = 8 - x^2$, and a spider is sitting at $(4, 0)$. At what point along the curve does the spider first see the fly?

22. Compute the limit, without using L'Hospital's Rule. $\lim\limits_{x\to 7} \dfrac{\sqrt{x+2} - 3}{x - 7}$

23. For what value(s) of c does $f(x) = cx^4 - 2x^2 + 1$ have both a local maximum and a local minimum?

24. If $f(x) = \sqrt{1 - 2x}$, determine $f'(x)$ *by using the definition of the derivative.*

25. A *point of inflection* for a function f is the x value for which $f''(x)$ changes sign (either from positive to negative or vice versa).

    Find constants a and b so that $(1, 6)$ is an inflection point for $y = x^3 + ax^2 + bx + 1$.

    Hint: The IVT might come in handy

26. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f(3) = 2$ and $f'(6) = 7$. Find $F'(3)$.

27. Find the dimensions of the rectangle of largest area that has its base on the $x-$axis and the other two vertices on the parabola $y = 8 - x^2$.

28. Let $G(x) = h(\sqrt{x})$. Then where is G differentiable? Find $G'(x)$.

29. If position is given by: $f(t) = t^4 - 2t^3 + 2$, find the times when the acceleration is zero. Then compute the velocity at these times.

30. If $y = \sqrt{5t - 1}$, compute y'''.

31. Find a second degree polynomial so that $P(2) = 5$, $P'(2) = 3$, and $P''(2) = 2$.

32. Find a function $f(x)$ so that $f'(x) = 4 - 3(1 + x^2)^{-1}$, and $f(1) = 0$

33. If $f(x) = (2 - 3x)^{-1/2}$, find $f(0)$, $f'(0)$, $f''(0)$.

34. Car A is traveling west at 50 mi/h, and car B is traveling north at 60 mi/h. Both are headed for the intersection between the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

35. Compute Δy and dy for the value of x and Δx: $f(x) = 6 - x^2$, $x = -2$, $\Delta x = 0.4$.

36. Find the linearization of $f(x) = \sqrt{1 - x}$ at $x = 0$.

37. Find $f(x)$, if $f''(x) = t + \sqrt{t}$, and $f(1) = 1$, $f'(1) = 2$.

38. Find $f'(x)$ directly from the definition of the derivative (using limits and without L'Hospital's rule):

    (a) $f(x) = \sqrt{3 - 5x}$

    (b) $f(x) = x^2$

(c) $f(x) = x^{-1}$

39. If $f(0) = 0$, and $f'(0) = 2$, find the derivative of $f(f(f(f(x))))$ at $x = 0$.

40. Differentiate:

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ -\sqrt{-x} & \text{if } x < 0 \end{cases}$$

Is f differentiable at $x = 0$? Explain.

41. $f(x) = |\ln(x)|$. Find $f'(x)$.

42. $f(x) = xe^{g(\sqrt{x})}$. Find $f'(x)$.

43. Find a formula for dy/dx: $x^2 + xy + y^3 = 0$.

44. Show that 5 is a critical number of $g(x) = 2+(x-5)^3$, but that g does not have a local extremum there.

45. Find the general antiderivative:

(a) $f(x) = 4 - x^2 + 3e^x$

(b) $f(x) = \frac{3}{x^2} + \frac{2}{x} + 1$

(c) $f(x) = \frac{1+x}{\sqrt{x}}$

46. Find the slope of the tangent line to the following at the point (3,4): $x^2 + \sqrt{y}x + y^2 = 31$

47. Find the critical values: $f(x) = |x^2 - x|$

48. Does there exist a function f so that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all x?

49. Find a function f so that $f'(x) = x^3$ and $x+y = 0$ is tangent to the graph of f.

50. Find dy if $y = \sqrt{1-x}$ and evaluate dy if $x = 0$ and $dx = 0.02$. Compare your answer to Δy

51. Fill in the question marks: If f'' is positive on an interval, then f' is ? and f is ?.

52. If $f(x) = x - \cos(x)$, x is in $[0, 2\pi]$, then find the value(s) of x for which

(a) $f(x)$ is greatest and least.

(b) $f(x)$ is increasing most rapidly.

(c) The slopes of the lines tangent to the graph of f are increasing most rapidly.

53. Show there is *exactly* one solution to: $\ln(x) = 3 - x$.

54. Approximate the change in volume of a cone, if we assume the height to be constant and r changes from 2 to 2.1. ($V = \frac{1}{3}\pi r^2 h$)

55. Sketch the graph of a function that satisfies all of the given conditions:

$$f(1) = 5 \qquad f(4) = 2 \qquad f'(1) = f'(4) = 0$$
$$\lim_{x \to 2^+} f(x) = \infty, \qquad \lim_{x \to 2^-} f(x) = 3 \qquad f(2) = 4$$

56. If $s^2t + t^3 = 1$, find $\frac{dt}{ds}$ and $\frac{ds}{dt}$.

57. Find the specific antiderivative:

(a) $f'(x) = 3\sqrt{x} - \frac{1}{\sqrt{x}}, \quad f(1) = 2$

(b) $f''(x) = x^2 + 3\cos(x), \quad f(0) = 2, f'(0) = 3$

(c) $f''(x) = 3e^x + 5\sin(x), \quad f(0) = 1, f'(0) = 2$

(d) $f'(x) = \frac{4}{\sqrt{1-x^2}}, \quad f(1/2) = 1.$

58. Find the area of the largest rectangle that can be inscribed in a right triangle with legs measuring 3 cm and 4 cm, and two sides of the rectangle lie along the legs.

59. A piece of wire 10 m long is cut into two pieces-one piece is bent into the shape of a square, the other into an equilateral triangle. How should the wire be cut so that the total enclosed area is a minimum?

60. A kite 100 feet above the ground moves horizontally (it stays 100 feet above the ground) away from you at a rate of 8 ft/sec. At what rate is the angle between the string and the ground decreasing when 200 feet of string has been let out?