# Extra Practice: Algebra and Trig

1. Perform each of the following operations:

   (a) $\dfrac{3s^2 - 48}{s^2 + 2s - 8} \div \dfrac{7s - 28}{s^2 - 4s + 4}$

   (b) $\dfrac{\frac{1}{p} + \frac{1}{q}}{1 - \frac{1}{pq}}$

2. Simplify each of the following:

   (a) $\dfrac{(p+1)^{1/2} - p(1/2)(p+1)^{-1/2}}{p+1}$

   (b) $\dfrac{3(2x^2+5)^{1/3} - x(2x^2+5)^{-2/3}(4x)}{(2x^2+5)^{2/3}}$

   (c) $\dfrac{(r-2)^{2/3} - r(2/3)(r-2)^{-1/2}}{(r-2)^{4/3}}$

3. Practice with rules of exponents: Simplify, writing each expression without negative exponents:

   (a) $\dfrac{6r^3s^{-2}}{6^{-1}r^4s^{-3}}$

   (b) $\dfrac{(2x^{-3})^2(3x^2)^{-2}}{6(x^2y^3)^{-1}}$

4. Practice with the inverse trigonometric functions (See Section 1.6)

   (a) Find the exact value of each expression:

   $$\cos^{-1}(-1) \qquad \arctan(1) \qquad \sin^{-1}(1/\sqrt{2}) \qquad \tan^{-1}(1/\sqrt{3})$$

   (b) Show (using a triangle) that $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$, then simplify $\tan(\cos^{-1}(x))$ using a similar technique.

5. Solve for $x$:

   (a) $2\cos(x) + 1 > 0$ (in the interval $[0, 2\pi]$)

   (b) $(x-2)/(x^2 - 2x - 3) \le 0$

   (c) $\sin(x) > \cos(x)$ (in the interval $[0, 2\pi]$)

# Solutions

1. Perform each of the following operations:

   (a) $\dfrac{7}{3(s-2)}$

   (b) $\dfrac{q+p}{pq-1}$

2. Simplify each of the following:

   (a) $\dfrac{3p+2}{2(p+1)^{3/2}}$

   (b) $\dfrac{2x^2+15}{(2x^2+5)^{4/3}}$

   (c) $\dfrac{r-6}{3(r-2)^{5/3}}$

3. Practice with rules of exponents: Simplify, writing each expression without negative exponents:

   (a) $\dfrac{36s}{r}$

   (b) $\dfrac{2y^3}{27x^8}$

4. Practice with the inverse trigonometric functions (See Section 1.6)

   (a) Find the exact value of each expression:

   $$\cos^{-1}(-1) = \pi \qquad \arctan(1) = \frac{\pi}{4} \qquad \sin^{-1}(1/\sqrt{2}) = \frac{\pi}{4} \qquad \tan^{-1}(1/\sqrt{3}) = \frac{\pi}{6}$$

   (b) Show (using a triangle) that $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$, then simplify $\tan(\cos^{-1}(x))$ using a similar technique.
   SOLUTION: For the first one, use $\theta = \sin^{-1}(x)$, or $\sin(\theta) = x$ and draw the appropriate right triangle. The hypotenuse is 1, the legs are $x$ and (by the Pythagorean Theorem) $\sqrt{1-x^2}$, so the cosine of the given angle is $\sqrt{1-x^2}/1$.
   Similarly, the right triangle for the second one label $\theta, x$ and 1 so that $\cos(\theta) = x/1$, then the other leg is $\sqrt{1-x^2}$. Take the tangent of $\theta$ to get $\sqrt{1-x^2}/x$.

5. Solve for $x$:

   (a) $2\cos(x) + 1 > 0$
   SOLUTION: $\cos(x) = -1/2$ if $x = 2\pi/3$ or $x = 4\pi/3$ (on the unit circle). Between these angles, $\cos(x) > -1/2$, so: $0 < x < 2\pi/3$ or $4\pi/3 < x < 2\pi$.

(b) $(x-2)/(x^2-2x-3) \leq 0$ Use a sign chart. The factors are zero where $x = -1, 2$, and 3, which divides the number line into four parts. The ones in which the expression is negative are:

$$x < -1 \quad \text{or} \quad 2 \leq x < 3$$

(c) $\sin(x) > \cos(x)$
SOLUTION: Using $\theta$, and the unit circle, we look at points $x = \cos(\theta)$ and $y = \sin(\theta)$ where $y > x$- They are equal where the circle and the line $y = x$ meet- At $\theta = \pi/4$ and $5\pi/4$, and $y > x$ in between:

$$5\pi/4 < \theta < \pi/4$$