Group Exercise SOLUTIONS

1. Solve the equation for x:

   (a) $\left|\dfrac{2x-1}{x+1}\right| = 3$ SOLUTION: From the definition of the absolute value, we have two equations to solve:

   $$\frac{2x-1}{x+1} = 3 \qquad \frac{2x-1}{x+1} = -3$$

   Solve these to get:

   $$\begin{aligned} 2x-1 &= 3x+3 \\ x &= -4 \end{aligned} \qquad \begin{aligned} 2x-1 &= -3x-3 \\ x &= -2/5 \end{aligned}$$

   Be sure to check your answer. In this case, both solutions are valid since the denominator is not zero.

   (b) $|x+3| = |2x+1|$ SOLUTION: We actually have 4 equations, but they boil down to 2 equations:

   $$\text{LHS} = \text{RHS} \qquad \text{LHS} = -\text{RHS} \qquad -\text{LHS} = \text{RHS} \qquad -\text{LHS} = -\text{RHS}$$

   So we just solve the second two:

   $$x+3 = 2x+1 \quad \Rightarrow \quad x = 2 \qquad\qquad x+3 = -2x-1 \quad \Rightarrow \quad 3x = -4 \Rightarrow x = -\frac{4}{3}$$

2. Solve the inequality x:

   (a) $1 < 4 - 2x \leq 5$

   SOLUTION: We could break this up, but let's see what we get first.

   $$-3 < -2x \leq 1 \quad \Rightarrow \quad -\frac{1}{2} \leq x < \frac{3}{2}$$

   (b) $x^3 + 3x < 4x^2$

   SOLUTION: Factor first

   $$x^3 - 4x^2 + 3x < 0 \quad \Rightarrow \quad x(x^2 - 4x + 3) < 0 \quad \Rightarrow \quad x(x-3)(x-1) < 0$$

   Now do the sign analysis:

	$x<0$	$0<x<1$	$1<x<3$	$x>3$
x	$-$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$
$x-1$	$-$	$-$	$+$	$+$
	$-$	$+$	$-$	$+$

   Therefore, $0 < x < 1$ or $x > 3$.

(c) $\dfrac{(x-1)(2-x)}{(x+1)} \geq 0$

SOLUTION: This is ready for the sign analysis:

	$x < -1$	$-1 < x < 1$	$1 < x < 2$	$x > 2$
$x+1$	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$+$	$+$
$2-x$	$+$	$+$	$+$	$-$
	$+$	$-$	$+$	$-$

Therefore, $x < -1$ or $1 < x < 2$.

3. Find an equation of the line that satisfies the given condition:

   (a) Through $(2, -3)$ perpendicular to $2x + 5y + 8 = 0$

   SOLUTION: The slope of the given line is $-2/5$, so the slope of our line is $5/2$: $y + 3 = (5/2)(x - 1)$

   (b) Through $(2, -3)$ parallel to the $y-$axis.

   SOLUTION: Parallel to the $y-$axis means that it is a vertical line (slope is undefined): $x = 2$

   (c) Through $(-3, -5)$ with a slope of 6: $y + 5 = 6(x + 3)$

   (d) Perpendicular to the previous line, through $(1, 1)$: $y - 1 = \frac{-1}{6}(x - 1)$