

Summary: To Exam 1 (Up to 2.8)

General Background: Chapter 1 and Appendices

There is a lot of algebra and trigonometry in Chapter 1, and Appendices A, B, C and D, so this is not an exhaustive list of everything you need to know, but there are some things we highlighted:

1. Construct the equation of a line (pt-slope form), Use the quadratic formula, find trigonometric values from right triangles/unit circle, function composition, simplify exponentials/logs using rules of exponents/logs.
2. Definitions: $|x|$, “one-to-one”, “natural domain”
3. Be able to “Find the domain”.
4. Use a **sign chart** to determine where an expression is positive/negative.
5. Know the difference between “inverse of a function” and the reciprocal of a function.
6. Given a formula for $f(x)$, be able to compute expressions like $f(2+h)$.

The Limit

1. Be able to compute limits algebraically and graphically.
2. Understand the meaning of, and be able to compute right and left-hand limits (algebraically and graphically)
3. Work with and understand the definition of the limit
 - Word definition: $\lim_{x \rightarrow a} f(x) = L$ means that we can keep the $f(x)$ values arbitrarily close to L by keeping the x -values sufficiently close to a .
 - Be able to state ϵ, δ definition: $\lim_{x \rightarrow a} f(x) = L$ means that, for every $\epsilon > 0$, there is a $\delta > 0$ such that
$$\text{if } 0 < |x - a| < \delta, \quad \text{then } |f(x) - L| < \epsilon$$
 - Show what the ϵ, δ definition means graphically and algebraically (for “simple” functions).
4. Algebraic Methods:
 - (a) Simplify (e.g., absolute values)
 - (b) Factor and Cancel
 - (c) Multiply by Conjugate
 - (d) Divide by x^n (Mainly for $x \rightarrow \infty$)
Be careful! $x = \sqrt{x^2}$ if $x \geq 0$, but if $x < 0$, $x = -\sqrt{x^2}$
5. The Squeeze Theorem.
6. Horizontal/Vertical Asymptotes:
 - (a) $x = a$ is a vertical asymptote for $f(x)$ if one of the following limits is infinite: $\lim_{x \rightarrow a^\pm} f(x)$
 - (b) $y = b$ is a horizontal asymptote for $f(x)$ if one of the following is true: $\lim_{x \rightarrow \pm\infty} f(x) = b$ Our template function:

$$\lim_{x \rightarrow \pm\infty} \frac{1}{x^r} = 0, \quad r > 0$$

(Note: x^r needs to be computable if $x \rightarrow -\infty$) Also, in a similar vein:

$$\lim_{x \rightarrow \infty} e^{-x} = 0$$

The inverse tangent has horizontal asymptotes:

$$\lim_{x \rightarrow \pm\infty} \tan^{-1}(x) = \pm \frac{\pi}{2}$$

And in general, if a function has a vertical asymptote at $x = a$, its inverse function will have a horizontal asymptote at $y = a$.

7. Intuition that can be used:

- (a) “ $\infty + \infty = \infty$ ”, but $\infty - \infty$ is not necessarily 0.
- (b) If the denominator goes to zero, but the numerator does not, the limit is $\pm\infty$.
- (c) If the denominator goes to $\pm\infty$, and the numerator does not, the overall limit goes to zero.
- (d) Given a rational function, if the degree of the numerator is larger than the denominator, the function goes to $\pm\infty$ as $x \rightarrow \pm\infty$.

If the degree of the denominator is larger, then the function goes to zero (again, as $x \rightarrow \pm\infty$)

8. Limit Laws (Sect 1.3)

Continuity

1. Definition: f is continuous at $x = a$ if $\lim_{x \rightarrow a} f(x) = f(a)$

This means 3 things: (1) $f(a)$ exists, (2) The limit exists, and (3) Items 1 and 2 are the same number.

2. Show that a function is not continuous at a point by stating which of the three parts are violated.

3. Show that a function is continuous by using the definition.

4. Give the meaning of “continuous from the right” and “continuous from the left”.

5. Theory about continuous functions: Know that our usual functions (see the list in Theorem 7) are all continuous *on their domain*. Know that the sum/difference, product/quotient of continuous functions is continuous (with a possibly restricted domain)

6. Be able to state and use the **Intermediate Value Theorem**:

If f is continuous on $[a, b]$, and N is a number between $f(a)$ and $f(b)$, there is at least one c in $[a, b]$ so that $f(c) = N$.

In practice, we usually use the IVT as:

If f is continuous, and $f(x_1) > 0, f(x_2) < 0$, then there is a c between x_1 and x_2 where $f(c) = 0$ (f has at least one root in the interval between x_1 and x_2).

The Derivative

1. Know the definition of Average Velocity and the technique we use to get Instantaneous Velocity (aka Velocity)

2. Definition: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ Be able to compute this given numerical values of a , or as an arbitrary value of a (you would be given $f(x)$).

3. Interpretations of the Derivative of f at $x = a$:

- (a) The velocity at $x = a$.
- (b) The slope of the tangent line at $(a, f(a))$.
- (c) The instantaneous rate of change of f at $x = a$.

4. Equation of the Tangent Line at $x = a$: This is the line going through $(a, f(a))$ with slope $f'(a)$. The best (and fastest) way to write the line: $y - f(a) = f'(a)(x - a)$

5. Be able to compute the derivative using the definition.