Calculus I Review Questions

NOTE: You should also look over your old exams, old review sheets, quizzes and homework.

1. Finish the definition:
   (a) $\lim_{x\to a} f(x) = L$ means that, for every ...
   (b) A function f is continuous at $x = a$ if:
   (c) $f'(x) =$
2. What are the three "Value" Theorems (state each):
3. Find the point on the parabola $x + y^2 = 0$ that is closest to the point $(0, -3)$.
4. Write the equation of the line tangent to
$$x = \sin(2y)$$
at $x = 1$.
5. For what value(s) of A, B, C will $y = Ax^2 + Bx + C$ satisfy the differential equation
$$\frac{1}{2}y'' - 2y' + y = 3x^2 + 2x + 1$$
6. For what value(s) of k will $y = \mathrm{e}^{kt}$ satisfy the differential equation
$$y'' + 4y' + 3y = 0$$
7. Compute the derivative of y with respect to x:
   (a) $y = \sqrt[3]{2x+1}\,\sqrt[5]{3x-2}$
   Use logarithmic differentiation
   (b) $y = \frac{1}{1+u^2}$, where $u = \frac{1}{1+x^2}$
   (c) $\sqrt[3]{y} + \sqrt[3]{x} = 4xy$
   (d) $\sqrt{x+y} = \sqrt[3]{x-y}$
   (e) $y = \sin(2\cos(3x))$
   (f) $y = (\cos(x))^{2x}$
   (g) $y = \sin^{-1}(3x) + 4^{3x} + \frac{x}{x+2}$
   (h) $y = \log_3(x^2 - x)$
   (Rewrite the expression first)
   (i) $y = \cot(3x^2 + 5)$
   (j) $\sqrt{x} + \sqrt[3]{y} = 1$
   (k) $x\tan(y) = y - 1$
   (l) $y = \frac{-2}{\sqrt[4]{t^3}}$, where $t = \ln(x^2)$.
   (m) $y = 3^{-1/x}$
8. Find the local maximums and minimums: $f(x) = x^3 - 3x + 1$ Show your answer is correct by using both the first derivative test and the second derivative test.
9. Compute the limit, if it exists. You may use any method (except a numerical table).
   (a) $\lim_{x\to 0} \frac{x - \sin(x)}{x^3}$
   (b) $\lim_{x\to 0} \frac{1 - \mathrm{e}^{-2x}}{\sec(x)}$
   (c) $\lim_{x\to 4^+} \frac{x-4}{|x-4|}$
   (d) $\lim_{x\to -\infty} \sqrt{\frac{2x^2-1}{x+8x^2}}$
   (e) $\lim_{x\to\infty} \sqrt{x^2+x+1} - \sqrt{x^2-x}$
   (f) $\lim_{h\to 0} \frac{(1+h)^{-2} - 1}{h}$
   (g) $\lim_{x\to\infty} x^3\mathrm{e}^{-x^2}$
   (h) $\lim_{x\to 1} \frac{x^{1000} - 1}{x - 1}$
   (i) $\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)}$
   (j) $\lim_{x\to 1} x^{\frac{1}{1-x}}$

10. Determine all vertical/horizontal asymptotes and critical points of $f(x) = \frac{2x^2}{x^2-x-2}$

11. Find values of m and b so that (1) f is continuous, and (2) f is differentiable.

$$f(x) = \begin{cases} x^2 & \text{if} \quad x \le 2 \\ mx+b & \text{if} \quad x > 2 \end{cases}$$

12. Find the local and global extreme values of $f(x) = \frac{x}{x^2+x+1}$ on the interval $[-2, 0]$.

13. Suppose f is differentiable so that:

$$f(1) = 1,\ f(2) = 2,\ f'(1) = 1\ f'(2) = 2$$

If $g(x) = f(x^3 + f(x^2))$, evaluate $g'(1)$.

14. Let $x^2y + a^2xy + \lambda y^2 = 0$

    (a) Let a and λ be constants, and let y be a function of x. Calculate $\frac{dy}{dx}$:

    (b) Let x and y be constants, and let a be a function of λ. Calculate $\frac{da}{d\lambda}$:

15. Show that $x^4 + 4x + c = 0$ has at most one solution in the interval $[-1, 1]$.

16. True or False, and give a short explanation.

    (a) If $f(x)$ is decreasing and $g(x)$ is decreasing, then $f(x)g(x)$ is decreasing.

    (b) $\frac{d}{dx}(10^x) = x10^{x-1}$

    (c) If $f'(x)$ exists and is nonzero for all x, then $f(1) \neq f(0)$.

    (d) If $2x + 1 \le f(x) \le x^2 + 2$ for all x, then $\lim_{x\to 1} f(x) = 3$.

    (e) If $f'(r)$ exists, then $\lim\limits_{x\to r} f(x) = f(r)$

    (f) If f and g are differentiable, then: $\frac{d}{\mathrm{dx}}(f(g(x)) = f'(x)g'(x)$

    (g) If $f(x) = x^2$, then the equation of the tangent line at $x = 3$ is: $y - 9 = 2x(x - 3)$

    (h) $\lim\limits_{\theta\to\frac{\pi}{3}} \dfrac{\cos(\theta) - \frac{1}{2}}{\theta - \frac{\pi}{3}} = -\sin\left(\dfrac{\pi}{3}\right)$

    (i) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

    (j) $5^{\log_5(2x)} = 2x$, for $x > 0$.

    (k) $\frac{d}{dx}\ln(|x|) = \frac{1}{x}$, for all $x \neq 0$.

    (l) $\lim\limits_{x\to 1} \dfrac{x^2 + 6x - 7}{x^2 + 5x - 6} = \dfrac{\lim_{x\to 1} x^2 + 6x - 7}{\lim_{x\to 1} x^2 + 5x - 6}$

    (m) If neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists, then $\lim_{x\to a} f(x)g(x)$ does not exist.

    (n) $\frac{x^2-1}{x-1} = x + 1$

17. Find the domain of $\ln(x - x^2)$:

18. Find the value of c guaranteed by the Mean Value Theorem, if $f(x) = \frac{x}{x+2}$ on the interval $[1, 4]$.

19. Given that the graph of f passes through the point $(1, 6)$ and the slope of the tangent line at $(x, f(x))$ is $2x + 1$, find $f(2)$.

20. Simplify, using a triangle: $\cos(\tan^{-1}(x))$ (No calculus needed).

21. A fly is crawling from left to right along the curve $y = 8 - x^2$, and a spider is sitting at $(4, 0)$. At what point along the curve does the spider first see the fly?

22. Compute the limit, without using L'Hospital's Rule. $\lim\limits_{x\to 7} \dfrac{\sqrt{x+2} - 3}{x - 7}$

23. For what value(s) of c does $f(x) = cx^4 - 2x^2 + 1$ have both a local maximum and a local minimum?

24. Find constants a and b so that $(1, 6)$ is an inflection point for $y = x^3 + ax^2 + bx + 1$.

    Hint: The IVT might come in handy

25. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f(3) = 2$ and $f'(6) = 7$. Find $F'(3)$.

26. Find the dimensions of the rectangle of largest area that has its base on the $x-$axis and the other two vertices on the parabola $y = 8 - x^2$.

27. Let $G(x) = h(\sqrt{x})$. Then where is G differentiable? Find $G'(x)$.

28. If position is given by: $f(t) = t^4 - 2t^3 + 2$, find the times when the acceleration is zero. Then compute the velocity at these times.

29. If $y = \sqrt{5t - 1}$, compute y'''.

30. If $f(x) = (2 - 3x)^{-1/2}$, find $f(0)$, $f'(0)$, $f''(0)$.

31. Car A is traveling west at 50 mi/h, and car B is traveling north at 60 mi/h. Both are headed for the intersection between the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

32. Find the linearization of $f(x) = \sqrt{1 - x}$ at $x = 0$.

33. Find $f(x)$, if $f''(x) = t + \sqrt{t}$, and $f(1) = 1$, $f'(1) = 2$.

34. Find $f'(x)$ directly from the definition of the derivative (using limits and without L'Hospital's rule):
    (a) $f(x) = \sqrt{3 - 5x}$
    (b) $f(x) = x^2$
    (c) $f(x) = x^{-1}$

35. If $f(0) = 0$, and $f'(0) = 2$, find the derivative of $f(f(f(f(x))))$ at $x = 0$.

36. Differentiate:
$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ \sqrt{-x} & \text{if } x < 0 \end{cases}$$
    Is f differentiable at $x = 0$? Explain.

37. $f(x) = |\ln(x)|$. Find $f'(x)$.

38. $f(x) = xe^{g(\sqrt{x})}$. Find $f'(x)$.

39. Find a formula for dy/dx: $x^2 + xy + y^3 = 0$.

40. Show that 5 is a critical number of $g(x) = 2 + (x - 5)^3$, but that g does not have a local extremum there.

41. Find the general antiderivative:
    (a) $f(x) = 4 - x^2 + 3e^x$
    (b) $f(x) = \frac{3}{x^2} + \frac{2}{x} + 1$
    (c) $f(x) = \frac{1+x}{\sqrt{x}}$

42. Find the slope of the tangent line to the following at the point (3,4): $x^2 + \sqrt{y}x + y^2 = 31$

43. Find the critical values: $f(x) = |x^2 - x|$

44. Does there exist a function f so that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all x?

45. Find a function f so that $f'(x) = x^3$ and $x + y = 0$ is tangent to the graph of f.

46. Find dy if $y = \sqrt{1 - x}$ and evaluate dy if $x = 0$ and $dx = 0.02$. Compare your answer to Δy

47. Fill in the question marks: If f'' is positive on an interval, then f' is ? and f is ?.

48. If $f(x) = x - \cos(x)$, x is in $[0, 2\pi]$, then find the value(s) of x for which

   (a) $f(x)$ is greatest and least.

   (b) $f(x)$ is increasing most rapidly.

   (c) The slopes of the lines tangent to the graph of f are increasing most rapidly.

49. Show there is *exactly* one solution to: $\ln(x) = 3 - x$.

50. Sketch the graph of a function that satisfies all of the given conditions:

$$\begin{array}{ccc} f(1) = 5 & f(4) = 2 & f'(1) = f'(4) = 0 \\ \lim_{x\to 2^+} f(x) = \infty, & \lim_{x \to 2^-} f(x) = 3 & f(2) = 4 \end{array}$$

51. If $s^2 t + t^3 = 1$, find $\frac{dt}{ds}$ and $\frac{ds}{dt}$.

52. Rewrite the function as a piecewise defined function (which gets rid of the absolute value signs):

$$f(x) = \frac{|3x+2|}{3x+2} \qquad f(x) = \left| \frac{x-2}{(x+1)(x+2)} \right|$$

53. Find all values of c and d so that f is continuous at all real numbers:

$$f(x) = \begin{cases} 2x^2 - 1 & \text{if } x < 0 \\ cx + d & \text{if } 0 \le x \le 1 \\ \sqrt{x+3} & \text{if } x > 1 \end{cases}$$

54. Find $f(x)$:

   (a) $f'(x) = \frac{3x-1}{\sqrt{x}}$, $f(1) = 2$.

   (b) $f''(x) = 3e^x + 5\sin(x)$, $f(0) = 1$, $f'(0) = 2$.

55. A rectangle is to be inscribed between the $x-$axis and the upper part of the graph of $y = 8 - x^2$ (symmetric about the $y-$axis). For example, one such rectangle might have vertices: $(1,0), (1,7), (-1,7), (-1,0)$ which would have an area of 14. Find the dimensions of the rectangle that will give the largest area.

56. What is the smallest possible rea of the triangle that is cut off by the first quadrant and whose hypotenuse is tangent to the curve $y = 4 - x^2$ at some point?

57. You're standing with Elvis (the dog) on a straight shoreline, and you throw the stick in the water. Let us label as "A" the point on the shore closest to the stick, and suppose that distance is 7 meters. Suppose that the distance from you to the point A is 10 meters. Suppose that Elvis can run at 3 meters per second, and can swim at 2 meters per second. How far along the shore should Elvis run before going in to swim to the stick, if he wants to minimize the time it takes him to get to the stick?

58. A water tank in the shape of an inverted cone with a circular base has a base radius of 2 meters and a height of 4 meters. If water is being pumped into the tank at a rate of 2 cubic meters per minute, find the rate at which the water level is rising when the water is 3 meters deep. ($V = \frac{1}{3}\pi r^2 h$)

59. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 foot per second, how fast is the angle between the ground and the ladder changing when the bottom of the ladder is 6 feet from the wall?

60. If a snowball melts so that its surface area decreases at a rate of 1 square centimeter per minute, find the rate at which the diameter decreases when the diameter is 10 cm. (The surface area is $A = 4\pi r^2$)

61. A man walks along a straight path at a speed of 4 ft/sec. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?