Solutions: Group Work, Section 3.4

Notice how problems 1-4 and 5-7 build up as go along:

1. $f(x) = \sin(3x)$

   SOLUTION: $f'(x) = 3\cos(3x)$

2. $g(x) = (\sin(3x))^3 = \sin^3(3x)$

   SOLUTION: Note that $g(x) = (f(x))^3$, so we could write $g'(x) = 3(f(x))^2 f'(x)$, or more directly:
   $$g'(x) = 3\sin^2(3x) \cdot 3\cos(3x) = 9\sin^2(3x)\cos(3x)$$

3. $h(x) = \sin^3(3x) + 5x$

   SOLUTION: Again, we might see that $h(x) = g(x) + 5x$, so that $h'(x) = g'(x) + 5$, or more directly:
   $$h'(x) = 9\sin^2(3x)\cos(3x) + 5$$

4. $j(x) = \left[\sin^3(3x) + 5x\right]^5$

   SOLUTION: We could write $j(x) = (h(x))^5$, so $j' = 5(h(x))^4 h'(x)$, or:
   $$j'(x) = 5\left[\sin^3(3x) + 5x\right]^4 \left(9\sin^2(3x)\cos(3x) + 5\right)$$

5. $k(x) = x + \dfrac{1}{x} = x + x^{-1}$

   SOLUTION: $k'(x) = 1 - x^{-2}$

6. $l(x) = \sqrt{x + x^{-1}}$

   SOLUTION: We could write $l(x) = \sqrt{k(x)}$, so that $l'(x) = \frac{1}{2}(k(x))^{-1/2}k'(x)$, or more directly:
   $$l'(x) = \frac{1}{2}\left(x + x^{-1}\right)^{-1/2}(1 - x^{-2})$$

7. Similarly, $m(x) = \sqrt{x + x^{-1}}\,[\sin^3(3x) + 5x]^5$, or we could write $m(x) = l(x) \cdot j(x)$, so that by the product rule:
   $$m'(x) = l'(x)j(x) + l(x)j'(x)$$

   Or, directly we could write the whole thing out, but it takes two lines!
   $$m'(x) = \frac{1}{2}\left(x + x^{-1}\right)^{-1/2}(1 - x^{-2})\left[\sin^3(3x) + 5x\right]^5 +$$
   $$\sqrt{x + x^{-1}}\left(5\left[\sin^3(3x) + 5x\right]^4\left(9\sin^2(3x)\cos(3x) + 5\right)\right)$$

SOLUTIONS TO THE GRAPHICAL EXERCISES:

There are some approximations here for the derivative of f:

1. $h'(1) = f'(g(1))g'(1) = f'(3)g'(1) \approx \frac{3}{2} \cdot 3 = \frac{9}{4}$
2. $h'(0) = f'(g(0))g'(0) = f'(0)g'(0) \approx -\frac{1}{2} \cdot 3 = \frac{-3}{2}$
3. $h'(2) = f'(g(2))g'(2) = f'(6)g'(2)$ However, $g'(2)$ does not exist (so $h'(2)$ does not exist).