

I have not used my calculator on this examination except for arithmetic, trigonometric, logarithmic, and exponential functions. I certify that the work on this exam is my own and that I have not discussed the specific contents of this exam with anyone prior to taking it.

Initials:

Instructions: Calculators may be used (see above), but you may not use any notes, the text or your colleagues (please keep your eyes on your own work!). I will have scratch paper available for you if you need any- Please turn it all in with your exam, and I will recycle it. Remember- you are being tested over the ideas/techniques from Calculus; answers with no appropriate justification will receive no credit.

1. Short answer:

- (a) Finish the definition: The limit as x approaches a of $f(x)$ is L if, for every $\epsilon > 0$, there is a $\delta > 0$ such that:
- (b) Finish the definition: $f'(x) =$
- (c) True or False (and give a short reason): If f is continuous at $x = a$, then f is differentiable at $x = a$.

2. Find $f'(x)$ directly from the definition of the derivative (using limits and without using l'Hospital's rule):

$$f(x) = \sqrt{1+x}$$

3. Derive the formula for the derivative of $y = \sin^{-1}(x)$.

4. Find dy/dx (solve for it, if necessary):

- (a) $y = \sin^3(x^2 + 1) + \tan^{-1}(x)$
- (b) $y = 3^{1/x} + \sec(x)$
- (c) $\sqrt{x+y} = 4xy$

5. Find the limit, if it exists (you may use any technique from class):

- (a) $\lim_{x \rightarrow 0} \frac{1 - e^{-2x}}{\sec(x)}$
- (b) $\lim_{x \rightarrow 4^+} \frac{x - 4}{|x - 4|}$
- (c) $\lim_{x \rightarrow -\infty} \sqrt{\frac{2x^2 - 1}{x + 8x^2}}$
- (d) $\lim_{x \rightarrow \infty} \sqrt{x^2 + x + 1} - \sqrt{x^2 - x}$

6. (a) Find the general antiderivative of $f(x) = \frac{1+x}{\sqrt[3]{x}}$

(b) Find a function f so that $f'(x) = x^3$ and $x + y = 0$ is tangent to the graph of f .

(c) Find the displacement of a particle if the acceleration is $a(t) = t + \sqrt{t}$, with $v(1) = 2$ and $s(1) = 1$.

7. Given the graph of the derivative, $f'(x)$, below, answer the following questions:

- (a) Find all intervals on which f is increasing.
- (b) Find all intervals on which f is concave up.
- (c) Sketch a possible graph of f if we require that $f(0) = -1$.

8. A rectangle is to be inscribed between the x -axis and the upper part of the graph of $y = 8 - x^2$ (symmetric about the y -axis). For example, one such rectangle might have vertices: $(1, 0), (1, 7), (-1, 7), (-1, 0)$ which would have an area of 14. Find the dimensions of the rectangle that will give the largest area.
9. A man walks along a straight path at a speed of 4 ft/sec. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?
10. Estimate the change in the volume of a sphere ($V = \frac{4}{3}\pi r^3$) using differentials, if the *circumference* changes from 2 to 2.1. Give the relative change in volume as well.
11. Find all values of c and d so that f is continuous at all real numbers:

$$f(x) = \begin{cases} 2x^2 - 1 & \text{if } x < 0 \\ cx + d & \text{if } 0 \leq x \leq 1 \\ \sqrt{x+3} & \text{if } x > 1 \end{cases}$$

Be sure it is clear from your work that you understand the definition of continuity.