Sample Exam 2

See Sample Exam 1 for the lead-in information.

1. Short Answer:
   (a) True or False? $\dfrac{x^2-1}{x-1} = x+1$
   (b) If $f'(2)$ exists, then $\lim_{x\to 2} f(x) = f(2)$
   (c) If $f(x) = (2-3x)^{-1/2}$, find $f(0)$, $f'(0)$ and $f''(0)$.
   (d) Show that the $x^4 + 4x + c = 0$ has at most one solution in the interval $[-1, 1]$.
2. Find dy/dx (solve for it if necessary):
   (a) $y = xe^{g(\sqrt{x})}$ for some differentiable function g.
   (b) $y = x^2 + 4^{1/x} + \sin^{-1}(3x+1) + \sec(x^2+x)$
   (c) $x\tan(y) = y - 1$
   (d) $\sqrt{x} + \sqrt{y} = 1$
3. Find $f'(x)$ directly from the definition of the derivative (using limits and without l'Hospital's rule): $f(x) = x^{-1}$
4. Find the limit if it exists. You may use any method (except for a numerical table).
   (a) $\lim_{x\to\infty} \sqrt{9x^2+x} - 3x$
   (b) $\lim_{x\to\pi^-} \dfrac{\sin(x)}{1-\cos(x)}$
   (c) $\lim_{x\to 0} \dfrac{x}{\tan^{-1}(4x)}$
   (d) $\lim_{x\to 1} x^{1/(1-x)}$
5. (a) Find the general antiderivative $F(x)$, if $f(x) = 2x\sqrt{x} + \dfrac{3 + x + \sqrt[3]{x^4}}{x}$
   (b) A stone is dropped from the observation deck of the Space Needle which is 160 m above the ground[1].
      i. If the acceleration due to gravity is 9.8 m/s^2, with what velocity does it strike the ground?
      ii. If the stone is thrown downward with a speed of 5 m/s, how long does it take to reach the ground?
6. Exercise 55, Section 4.9, p. 349 (For the exam, I would give you the instructions/graph).
7. You're standing with Elvis (the dog) on a straight shoreline, and you throw the stick in the water. Let us label as "A" the point on the shore closest to the stick, and suppose that distance is 7 meters. Suppose that the distance from you to the point A is 10 meters. Suppose that Elvis can run at 3 meters per second, and can swim at 2 meters per second. How far along the shore should Elvis run before going in to swim to the stick, if he wants to minimize the time it takes him to get to the stick?
8. A water tank in the shape of an inverted cone with a circular base has a base radius of 2 meters and a height of 4 meters. If water is being pumped into the tank at a rate of 2 cubic meters per minute, find the rate at which the water level is rising when the water is 3 meters deep. ($V = \frac{1}{3}\pi r^2 h$)
9. Explain why the following is true (if it is): The function $f(x) = \sqrt{1+2x}$ can be well approximated by $(1+x)/3$ if x is approximately 8.
10. Find m and b so that f is continuous and differentiable:
$$f(x) = \begin{cases} x^2 \text{ if } x \le 2 \\ mx + b \text{ if } x > 2 \end{cases}$$
11. Find the absolute maximum and minimum of $f(x) = |x^2 - x|$ on the interval $[0, 2]$.

[1] Please don't drop stones off the Space Needle!