Sample Exam 3

See Sample Exam 1 for the lead-in information.

1. Short answer

   (a) Explain the Mean Value Theorem in terms of velocity:
   Side Note for study: You should know the theorems MVT, IVT and EVT, and give situations in which they are used or perhaps an illustration of each.

   (b) Suppose the line $y = 2x + 1$ is tangent to $f(x)$ at $x = 1$. If $x_1 = 1$ is the initial guess for the root to $f(x)$ using Newton's Method, what is x_2?

   (c) True or False, and give a short reason:

      i. $\frac{d}{dx}(10^x) = x10^{x-1}$

      ii. $\displaystyle\lim_{x\to 1} \frac{x^2+6x-7}{x^2+5x-6} = \frac{\lim_{x\to 1} x^2+6x-7}{\lim_{x\to 1} x^2+5x-6}$

      iii. If $f(x) = x^2$, then the equation of the normal line at $x = 3$ is: $y - 9 = \frac{-1}{2x}(x-3)$

2. Find dy/dx (solve for it if necessary):

   (a) $y = 3^{x^2-1} + (x^2 - 3x + 1)^5 + (x^2 - 1)^{\sin(x)}$

   (b) $y = \dfrac{1-2x}{\sqrt[3]{x^5}}$

   (c) $x\sin(y) + y\sin(x) = 1$

   (d) $y = \ln|\csc(3x) + \cot(3x)|$

3. Find all vertical and horizontal asymptotes of $f(x) = \dfrac{2x^2-2}{x^2-x-2}$

4. Derive the formula for the derivative of $y = \sec^{-1}(x)$:

5. Find $f'(1)$ using the definition of the derivative (using limits and you may not use l'Hospital's rule), if $f(x) = \dfrac{x}{x+1}$

6. Find the limit, if it exists (you may use any method from class):

   (a) $\displaystyle\lim_{x\to 3} \frac{\sqrt{x+6}-x}{x^3-3x^2}$

   (b) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{x^2-9}}{2x-6}$

   (c) $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{e^x-1}\right)$

   (d) $\displaystyle\lim_{x\to 0}(1-2x)^{1/x}$

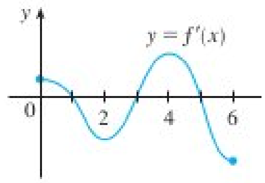
7. (a) A company estimates that the marginal cost (in dollars per item) of x items is $1.92 - 0.002x$.

      i. What is the marginal cost of 200 items, and how should we interpret that (that is, what does the marginal cost represent)?

ii. If the cost of producing one item is \$562, find the cost of producing 100 items.

(b) Find f if $f''(x) = 6x + \sin(x)$ if $f'(0) = 0$ and $f(0) = 3$.

8. Exercise 53, Section 4.9, p. 349.

9. The following is the graph of $f'(x)$:  (From exercise 6, section 4.3)

   (a) On what intervals is f increasing or decreasing?

   (b) On what intervals is f concave up or concave down?

   (c) At what points does f have a local maximum?

   (d) Sketch a graph of f''.

10. What is the smallest possible area of the triangle that is cut off by the first quadrant and whose hypotenuse is tangent to the curve $y = 4 - x^2$ at some point?

11. The period of oscillation of a pendulum is $P = 2\pi\sqrt{\frac{L}{32}}$, where L is the length of the pendulum. Estimate the change in P using differentials, if the length is changed from 2 to 2.1.

12. It has been estimated that since the second half of the 19th century, the population of the United States doubles approximately every 56 years. If the current population is approximately 311 million, when will the population reach half a billion?

    Side Note for Study: Some biologists like to use the model $y(t) = P_0 2^{kt}$ instead if $y(t) = P_0 e^{rt}$. Starting from the biologists' model, is there a formula that relates k and r?