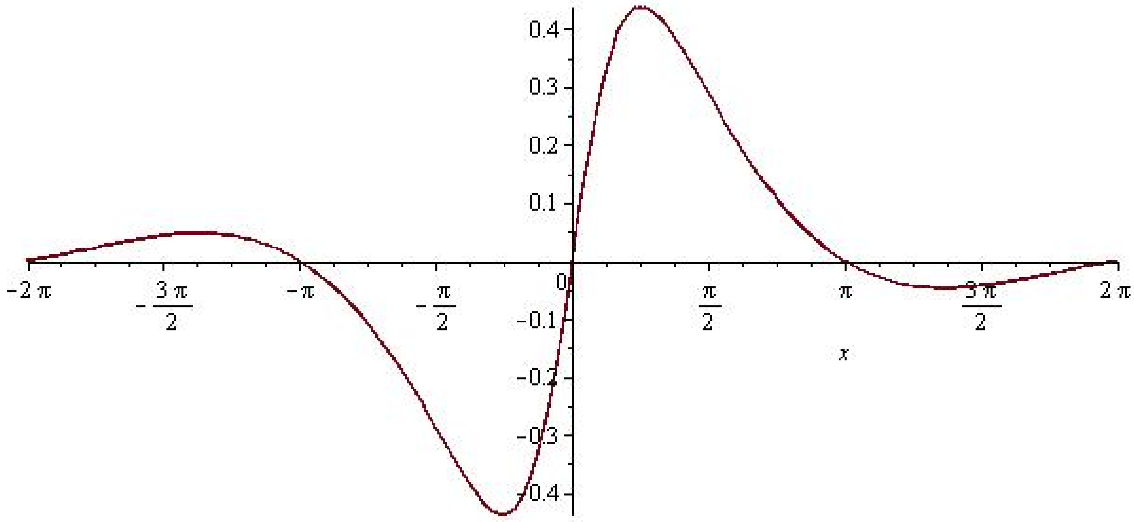
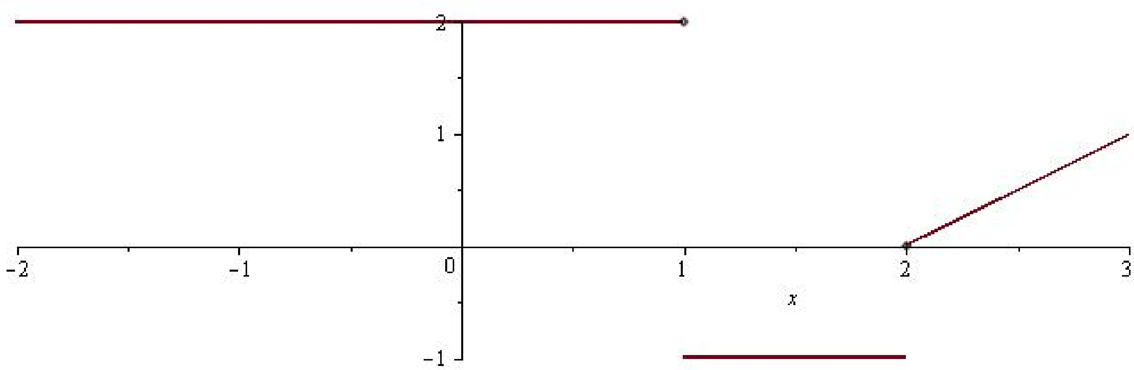
4.9 Extra Practice

1. Given the graph of $f'(x)$ below, sketch a graph of $f(x)$ (so that $f(0) = 0$).



2. A car is traveling at 100 km/h when the driver sees an accident 80 m (or 0.08 km) ahead and slams on the brakes. What constant deceleration is required to stop the car within the 80 m?

3. Given the graph of $f'(x)$ below, sketch a graph of $f(x)$ (so that $f(0) = 0$).

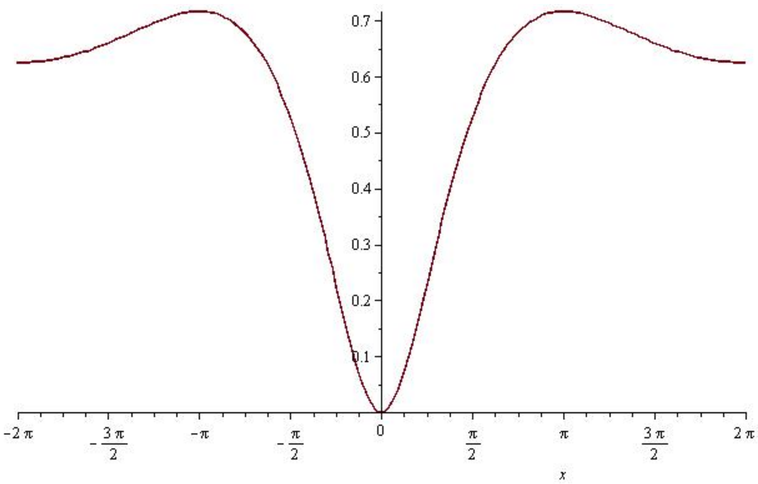


Extra: See if you can come up with a formula for the general antiderivative!

Solutions

For the graphical problems, be sure that you can identify intervals on which f is increasing/decreasing, and intervals on which f is concave up/concave down.

1. You should check for general shape; you can ignore the numerical values along the $y-$axis.



2. SOLUTION: We have to decide on the setup- Let's say that our current position is 0 at time 0, which would make $v(0) = 100$ (that makes the direction of travel positive). If we define the acceleration to be k km/h^2, then

$$a(t) = k \quad \Rightarrow v(t) = kt + C \quad \Rightarrow v(t) = kt + 100 \quad \Rightarrow \quad s(t) = \frac{1}{2}kt^2 + 100t$$

Now we need the time it takes for the velocity to reach 0:

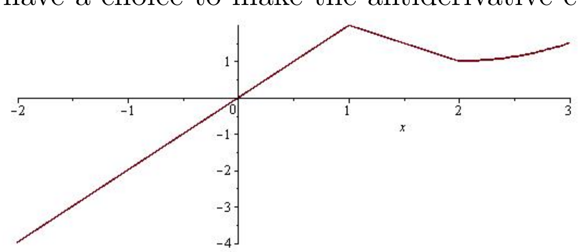
$$0 = kt + 100 \quad \Rightarrow \quad t = -\frac{100}{k}$$

And if we substitute that into $s(t)$, we want the result to be less than or equal to 0.08 km:

$$0.08 = \frac{1}{2}k\left(-\frac{100}{k}\right)^2 + 100\left(-\frac{100}{k}\right) = -\frac{5000}{k}$$

Therefore, $k \approx -62,500$km/h^2 (which is approx. -4.82 m/s^2)

3. You should have two line segments and a half of parabola. Notice that the two line segments (for $x < 1$, and $1 < x < 2$) did not have to touch, but it is common when we have a choice to make the antiderivative continuous.



FYI- The formula for the general antiderivative:

$$f(x) = \begin{cases} 2x + C_1 & \text{if } x < 1 \\ -x + C_2 & \text{if } 1 < x < 2 \\ \frac{1}{2}(x-2)^2 + C_3 & \text{if } x > 2 \end{cases}$$