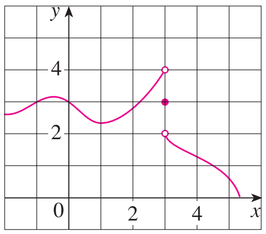
3. Explain the meaning of each of the following.

(a) $\lim_{x \to -3} f(x) = \infty$ (b) $\lim_{x \to 4^+} f(x) = -\infty$

4. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 0} f(x)$ (b) $\lim_{x \to 3^-} f(x)$ (c) $\lim_{x \to 3^+} f(x)$

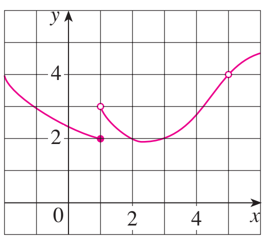
(d) $\lim_{x \to 3} f(x)$ (e) $f(3)$



5. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 1^-} f(x)$ (b) $\lim_{x \to 1^+} f(x)$ (c) $\lim_{x \to 1} f(x)$

(d) $\lim_{x \to 5} f(x)$ (e) $f(5)$



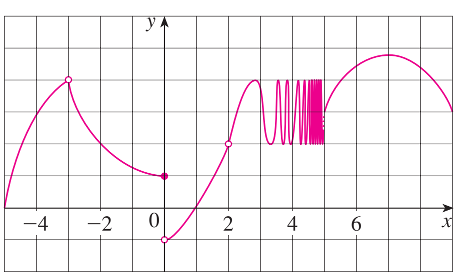
6. For the function h whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to -3^-} h(x)$ (b) $\lim_{x \to -3^+} h(x)$ (c) $\lim_{x \to -3} h(x)$

(d) $h(-3)$ (e) $\lim_{x \to 0^-} h(x)$ (f) $\lim_{x \to 0^+} h(x)$

(g) $\lim_{x \to 0} h(x)$ (h) $h(0)$ (i) $\lim_{x \to 2} h(x)$

(j) $h(2)$ (k) $\lim_{x \to 5^+} h(x)$ (l) $\lim_{x \to 5^-} h(x)$

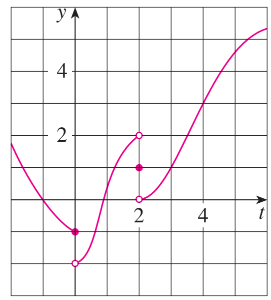


7. For the function g whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{t \to 0^-} g(t)$ (b) $\lim_{t \to 0^+} g(t)$ (c) $\lim_{t \to 0} g(t)$

(d) $\lim_{t \to 2^-} g(t)$ (e) $\lim_{t \to 2^+} g(t)$ (f) $\lim_{t \to 2} g(t)$

(g) $g(2)$ (h) $\lim_{t \to 4} g(t)$

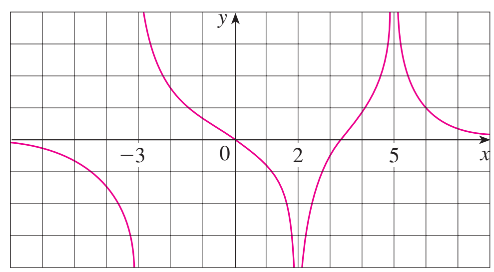


8. For the function R whose graph is shown, state the following.

(a) $\lim_{x \to 2} R(x)$ (b) $\lim_{x \to 5} R(x)$

(c) $\lim_{x \to -3^-} R(x)$ (d) $\lim_{x \to -3^+} R(x)$

(e) The equations of the vertical asymptotes.

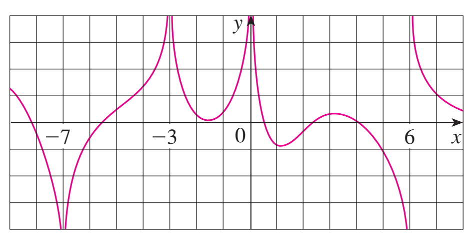


9. For the function f whose graph is shown, state the following.

(a) $\lim_{x \to -7} f(x)$ (b) $\lim_{x \to -3} f(x)$ (c) $\lim_{x \to 0} f(x)$

(d) $\lim_{x \to 6^-} f(x)$ (e) $\lim_{x \to 6^+} f(x)$

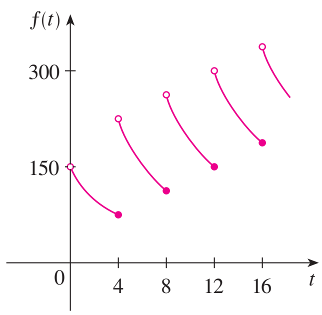
(f) The equations of the vertical asymptotes.



10. A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find

$$\lim_{t\to 12^-} f(t) \quad \text{and} \quad \lim_{t\to 12^+} f(t)$$

and explain the significance of these one-sided limits.



11. Use the graph of the function $f(x) = 1/(1 + e^{1/x})$ to state the value of each limit, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 0^-} f(x)$ (b) $\lim_{x\to 0^+} f(x)$ (c) $\lim_{x\to 0} f(x)$

12. Sketch the graph of the following function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists:

$$f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x & \text{if } -1 \leqslant x < 1 \\ (x-1)^2 & \text{if } x \geqslant 1 \end{cases}$$

13–16 Sketch the graph of an example of a function f that satisfies all of the given conditions.

13. $\lim_{x\to 1^-} f(x) = 2, \quad \lim_{x\to 1^+} f(x) = -2, \quad f(1) = 2$

14. $\lim_{x\to 0^-} f(x) = 1, \quad \lim_{x\to 0^+} f(x) = -1, \quad \lim_{x\to 2^-} f(x) = 0,$
$\lim_{x\to 2^+} f(x) = 1, \quad f(2) = 1, \quad f(0)$ is undefined

15. $\lim_{x\to 3^+} f(x) = 4, \quad \lim_{x\to 3^-} f(x) = 2, \quad \lim_{x\to -2} f(x) = 2,$
$f(3) = 3, \quad f(-2) = 1$

16. $\lim_{x\to 1} f(x) = 3, \quad \lim_{x\to 4^-} f(x) = 3, \quad \lim_{x\to 4^+} f(x) = -3,$
$f(1) = 1, \quad f(4) = -1$

17–20 Guess the value of the limit (if it exists) by evaluating the function at the given numbers (correct to six decimal places).

17. $\lim_{x\to 2} \dfrac{x^2 - 2x}{x^2 - x - 2}$, $\quad x = 2.5, 2.1, 2.05, 2.01, 2.005, 2.001,$
$1.9, 1.95, 1.99, 1.995, 1.999$

18. $\lim_{x\to -1} \dfrac{x^2 - 2x}{x^2 - x - 2}$,
$x = 0, -0.5, -0.9, -0.95, -0.99, -0.999,$
$-2, -1.5, -1.1, -1.01, -1.001$

19. $\lim_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$, $\quad x = \pm 1, \pm 0.5, \pm 0.1, \pm 0.05, \pm 0.01$

20. $\lim_{x\to 0^+} x \ln(x + x^2)$, $\quad x = 1, 0.5, 0.1, 0.05, 0.01, 0.005, 0.001$

21–24 Use a table of values to estimate the value of the limit. If you have a graphing device, use it to confirm your result graphically.

21. $\lim_{x\to 0} \dfrac{\sqrt{x+4} - 2}{x}$

22. $\lim_{x\to 0} \dfrac{\tan 3x}{\tan 5x}$

23. $\lim_{x\to 1} \dfrac{x^6 - 1}{x^{10} - 1}$

24. $\lim_{x\to 0} \dfrac{9^x - 5^x}{x}$

25–32 Determine the infinite limit.

25. $\lim_{x\to -3^+} \dfrac{x+2}{x+3}$

26. $\lim_{x\to -3^-} \dfrac{x+2}{x+3}$

27. $\lim_{x\to 1} \dfrac{2-x}{(x-1)^2}$

28. $\lim_{x\to 5^-} \dfrac{e^x}{(x-5)^3}$

29. $\lim_{x\to 3^+} \ln(x^2 - 9)$

30. $\lim_{x\to \pi^-} \cot x$

31. $\lim_{x\to 2\pi^-} x \csc x$

32. $\lim_{x\to 2^-} \dfrac{x^2 - 2x}{x^2 - 4x + 4}$

33. Determine $\lim_{x\to 1^-} \dfrac{1}{x^3 - 1}$ and $\lim_{x\to 1^+} \dfrac{1}{x^3 - 1}$

(a) by evaluating $f(x) = 1/(x^3 - 1)$ for values of x that approach 1 from the left and from the right,
(b) by reasoning as in Example 9, and
(c) from a graph of f.

34. (a) Find the vertical asymptotes of the function

$$y = \frac{x^2 + 1}{3x - 2x^2}$$

(b) Confirm your answer to part (a) by graphing the function.

35. (a) Estimate the value of the limit $\lim_{x\to 0} (1 + x)^{1/x}$ to five decimal places. Does this number look familiar?
(b) Illustrate part (a) by graphing the function $y = (1 + x)^{1/x}$.

36. (a) By graphing the function $f(x) = (\tan 4x)/x$ and zooming in toward the point where the graph crosses the y-axis, estimate the value of $\lim_{x\to 0} f(x)$.
(b) Check your answer in part (a) by evaluating $f(x)$ for values of x that approach 0.