

## Summary: To Exam 1 (Up through 2.7)

### General Background: Chapter 1 and Appendices

There is a lot of algebra and trigonometry in Chapter 1, and Appendices A, B, C and D, so this is not an exhaustive list of everything you need to know, but there are some things we highlighted:

1. Construct the equation of a line (pt-slope form), Use the quadratic formula, find trigonometric values from right triangles/unit circle, function composition, simplify exponentials/logs using rules of exponents/logs.
2. Definitions:  $|x|$ , “one-to-one”, “natural domain”
3. Be able to “Find the domain”.
4. Use a **sign chart** to determine where an expression is positive/negative.
5. Know the difference between “inverse of a function” and the reciprocal of a function.
6. Given a formula for  $f(x)$ , be able to compute expressions like  $f(2 + h)$ .

### The Limit

1. Be able to compute limits algebraically and graphically.
2. Understand the meaning of, and be able to compute right and left-hand limits.
3. Work with and understand the definition of the limit
  - Word definition:  $\lim_{x \rightarrow a} f(x) = L$  means that we can keep the  $f(x)$  values arbitrarily close to  $L$  by keeping the  $x$ -values sufficiently close to  $a$ .
  - Be able to state  $\epsilon, \delta$  definition:  $\lim_{x \rightarrow a} f(x) = L$  means that, for every  $\epsilon > 0$ , there is a  $\delta > 0$  such that
$$\text{if } 0 < |x - a| < \delta, \quad \text{then } |f(x) - L| < \epsilon$$
  - Show what the  $\epsilon, \delta$  definition means graphically (as in Exercises 1-4 of 2.4).
4. Algebraic Methods to compute limits:
  - (a) Simplify (e.g., absolute values)
  - (b) Factor and Cancel
  - (c) Multiply by Conjugate
  - (d) Divide by  $x^n$  (Mainly for  $x \rightarrow \infty$ ). Be careful!  $x = \sqrt{x^2}$  if  $x \geq 0$ , but if  $x < 0$ ,  $x = -\sqrt{x^2}$
5. The Squeeze Theorem.
6. Horizontal/Vertical Asymptotes:

(a)  $x = a$  is a vertical asymptote for  $f(x)$  if one of the following limits is infinite:  $\lim_{x \rightarrow a^\pm} f(x)$

(b)  $y = b$  is a horizontal asymptote for  $f(x)$  if one of the following is true:  $\lim_{x \rightarrow \pm\infty} f(x) = b$

Our template function:  $\lim_{x \rightarrow \pm\infty} \frac{1}{x^r} = 0, \quad r > 0$

(Note:  $x^r$  needs to be computable if  $x \rightarrow -\infty$ ) Also, in a similar vein:  $\lim_{x \rightarrow \infty} e^{-x} = 0$  The inverse tangent has horizontal asymptotes:

$$\lim_{x \rightarrow \pm\infty} \tan^{-1}(x) = \pm \frac{\pi}{2}$$

And in general, if a function has a vertical asymptote at  $x = a$ , its inverse function will have a horizontal asymptote at  $y = a$ .

7. Intuition that can be used:

- (a) “ $\infty + \infty = \infty$ ”, but  $\infty - \infty$  is not necessarily 0. (Similarly, the product but not the quotient)
- (b) If the denominator goes to zero, but the numerator does not, the limit is  $\pm\infty$ .
- (c) If the denominator goes to  $\pm\infty$ , and the numerator does not, the overall limit goes to zero.
- (d) Given a rational function, if the degree of the numerator is larger than the denominator, the function goes to  $\pm\infty$  as  $x \rightarrow \pm\infty$ .  
If the degree of the denominator is larger, then the function goes to zero (again, as  $x \rightarrow \pm\infty$ )

8. Limit Laws (Sect 1.3): Be able to summarize them (as on Quiz 3)

## Continuity

1. Definition:  $f$  is continuous at  $x = a$  if  $\lim_{x \rightarrow a} f(x) = f(a)$
2. Interpretation of the definition: This means 3 things: (1)  $f(a)$  exists, (2) The limit exists, and (3) Items 1 and 2 are the same number.
3. Show that a function is not continuous at a point by stating which of the three parts are violated.
4. Show that a function is continuous by using the definition.
5. Give the meaning of “continuous from the right” and “continuous from the left”.
6. Theory about continuous functions: Know that our usual functions (see the list in Theorem 7) are all continuous *on their domain*. Know that the sum/difference, product/quotient of continuous functions is continuous (with a possibly restricted domain)
7. Be able to state and use the **Intermediate Value Theorem**:

If  $f$  is continuous on  $[a, b]$ , and  $N$  is a number between  $f(a)$  and  $f(b)$ , there is at least one  $c$  in  $[a, b]$  so that  $f(c) = N$ .

In practice, we usually use the IVT as:

If  $f$  is continuous, and  $f(x_1) > 0$ ,  $f(x_2) < 0$ , then there is a  $c$  between  $x_1$  and  $x_2$  where  $f(c) = 0$  ( $f$  has at least one root in the interval between  $x_1$  and  $x_2$ ).

## The Derivative

1. Know the definition of Average Velocity and the technique we use to get Instantaneous Velocity (aka Velocity)
2. Definition:  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  Be able to compute this given numerical values of  $a$ , or as an arbitrary value of  $a$  (you would be given  $f(x)$ ).
3. Interpretations of the Derivative of  $f$  at  $x = a$ :
  - (a) The velocity at  $x = a$ .
  - (b) The slope of the tangent line at  $(a, f(a))$ .
  - (c) The instantaneous rate of change of  $f$  at  $x = a$ .
4. Equation of the Tangent Line at  $x = a$ : This is the line going through  $(a, f(a))$  with slope  $f'(a)$ . The best (and fastest) way to write the line:  $y - f(a) = f'(a)(x - a)$
5. Be able to compute the derivative using the definition.