Extra Practice: Optimization (4.7)

The following are some extra practice problems for "optimization", section 4.7. Solutions to be posted soon.

1. Suppose we have two numbers, one is a positive number and the other is its reciprocal. Find the two numbers so that the sum is small as possible.

   SOLUTION: Let x and $1/x$ be the two numbers. Then we want to find the minimum of:
   $$f(x) = x + \frac{1}{x} \qquad x \geq 0$$
   If we look at the critical points of f, we see that $x = \pm 1$ (but we only take $x = 1$). Further, the derivative is:
   $$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$$
   We see that $f'(x) < 0$ if $0 < x < 1$ and $f'(x) > 0$ if $x > 1$. By the first derivative test, $x = 1$ is the minimum.

2. Find two positive numbers such that their product is 16 and the sum is as small as possible.

3. A 20-inch piece of wire is bent into an L-shape. Where should the bend be made to minimize the distance between the two ends?

4. Find the point on the line $y = x$ closest to the point $(1, 0)$.

5. A box is constructed out of two different types of metal. The metal for the top and bottom, which are both square, costs \$1.00 per square foot, and the metal for the sides costs \$2.00 per square foot. Find the dimensions that minimize the cost of the box is the box must have a volume of 20 cubic feet.

6. A rectangle is to be inscribed between the $x-$axis and the upper part of the graph of $y = 8 - x^2$ (symmetric about the $y-$axis). For example, one such rectangle might have vertices: $(1, 0), (1, 7), (-1, 7), (-1, 0)$ which would have an area of 14. Find the dimensions of the rectangle that will give the largest area.

7. What is the smallest possible rea of the triangle that is cut off by the first quadrant and whose hypotenuse is tangent to the curve $y = 4 - x^2$ at some point?

8. You're standing with Elvis (the dog) on a straight shoreline, and you throw the stick in the water. Let us label as "A" the point on the shore closest to the stick, and suppose that distance is 7 meters. Suppose that the distance from you to the point A is 10 meters. Suppose that Elvis can run at 3 meters per second, and can swim at 2 meters per second. How far along the shore should Elvis run before going in to swim to the stick, if he wants to minimize the time it takes him to get to the stick?