

GROUP WORK 2, SECTION 3.1

Find the Error

It is a bright Spring morning. You have just finished your Chemistry lab, and have a Physics class starting in a half hour, so you have a little bit of time to sit on a park bench and relax by leafing through your *Calculus: Early Transcendentals* book. Suddenly, you notice a wild-eyed, hungry-looking stranger looking over your shoulder.

“Lies! Lies!” he yells. “That book there is filled with nothing but lies!”

“Why, you are mistaken,” you explain. “My *Calculus: Early Transcendentals* book is chock-a-block with knowledge and useful wisdom.”

“Oh yeah? Well what would your calculus book say about THIS?” he demands, and hands you a piece of paper with the following written on it:

$$\begin{aligned} \text{For } x > 0: \\ x &= \underbrace{1 + 1 + \cdots + 1}_{x \text{ times}} \\ x^2 &= \underbrace{x + x + \cdots + x}_{x \text{ times}} \\ D(x^2) &= D\left(\underbrace{x + x + \cdots + x}_{x \text{ times}}\right) \\ D(x^2) &= \underbrace{D(x) + D(x) + \cdots + D(x)}_{x \text{ times}} \\ 2x &= \underbrace{1 + 1 + \cdots + 1}_{x \text{ times}} \\ 2x &= x \\ 2 &= 1 \end{aligned}$$

“Put THAT in your pipe and smoke it!” At that, the gentleman runs off, screaming, “I’ll be back!” into the wind.

Is all of mathematics wrong? Is two really equal to one? Are “two for one” specials really no bargain at all? Is “six of one” really not “half a dozen of the other”? Or is there a mistake in your new friend’s reasoning? If so, what is it?