M125
Fall 2019

Sample Final 1

Name:______________

I certify that the work on this exam is my own and that I have not discussed the specific contents of this exam with anyone prior to taking it.

Initials:

Instructions: You may not use any notes, the text or your colleagues (please keep your eyes on your own work!). Remember- you are being tested over the ideas/techniques from Calculus; answers with no appropriate justification will receive no credit. There are some formulas at the end of the exam you might find helpful.

1. Give the definition of the derivative: $f'(x) =$
2. State the Fundamental Theorem of Calculus. It begins by having f be a continuous function on $[a, b]$.
3. True or False (and give a short reason):
   (a) If f is continuous at $x = a$, then f is differentiable at $x = a$.
   (b) If $3 \leq f(x) \leq 5$ for all x, then $6 \leq \int_1^3 f(x)\,dx \leq 10$.
   (c) All continuous functions have antiderivatives.
   (d) $\int_{-2}^{1} -x^{-2}\,dx = x^{-1}\big|_{-2}^{1} = \frac{3}{2}$
4. Find $f'(x)$ directly from the definition of the derivative $f(x) = \sqrt{1+x}$.
5. Derive the formula for the derivative of $y = \sin^{-1}(x)$.
6. Find dy/dx (solve for it, if necessary):
   (a) $y = \sin^3(x^2+1) + \tan^{-1}(x)$ (b) $y = 3^{1/x} + \sec(x)$ (c) $\sqrt{x+y} = 4xy$
7. Find the limit, if it exists (you may use any technique from class):
   (a) $\lim_{x\to 0} \frac{1 - e^{-2x}}{\sec(x)}$
   (b) $\lim_{x\to 4^+} \frac{x-4}{|x-4|}$
   (c) $\lim_{x\to -\infty} \sqrt{\frac{2x^2-1}{x+8x^2}}$
   (d) $\lim_{x\to 0^+} x^x$
8. Evaluate the Riemann sum by first writing it as an appropriate definite integral: $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1 + \frac{3i}{n}}$.
9. Differentiate: $F(x) = \int_{\sqrt{x}}^{x^2} \frac{t}{1+t}\,dt$
10. Evaluate the definite integral by using the definition. That is, first write the Riemann sum, then take the appropriate limit. (You must use the Riemann sum to get credit)
    $\int_0^3 1 + 3x\,dx$
11. Use an appropriate substitution to evaluate: $\int x^2 e^{x^3}\,dx$

12. Evaluate, or find the general indefinite integral.

(a) $\displaystyle\int \sqrt{x^3} + \frac{1}{x^2+1}\,dx$ (b) $\displaystyle\int_{-1}^{1} t(1-t)\,dt$ (c) $\displaystyle\int_{0}^{1} 5x - 5^x\,dx$

13. Evaluate:

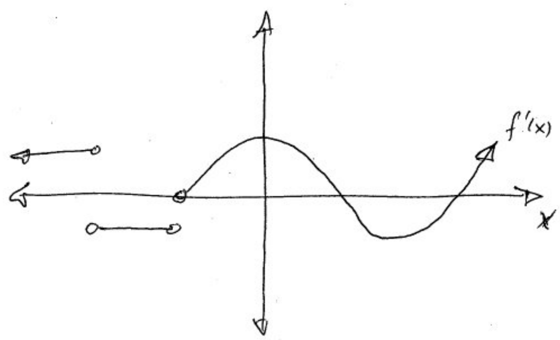
(a) $\displaystyle\int_0^1 \frac{d}{dx}\left(e^{\tan^{-1}(x)}\right)dx$ (b) $\displaystyle\frac{d}{dx}\int_0^1 e^{\tan^{-1}(x)}\,dx$ (c) $\displaystyle\frac{d}{dx}\int_0^x e^{\tan^{-1}(t)}\,dt$

14. Given the graph of the derivative, $f'(x)$, below, answer the following questions:

(a) Find all intervals on which f is increasing.

(b) Find all intervals on which f is concave up.

(c) Sketch a possible graph of f if we require that $f(0) = -1$.



15. A rectangle is to be inscribed between the x−axis and the upper part of the graph of $y = 8 - x^2$ (symmetric about the y−axis). For example, one such rectangle might have vertices: $(1,0), (1,7), (-1,7), (-1,0)$ which would have an area of 14. Find the dimensions of the rectangle that will give the largest area.

16. Find all values of c and d so that f is continuous at all real numbers:

$$f(x) = \begin{cases} 2x^2 - 1 & \text{if } x < 0 \\ cx + d & \text{if } 0 \le x \le 1 \\ \sqrt{x+3} & \text{if } x > 1 \end{cases}$$

Be sure it is clear from your work that you understand the definition of continuity.

Formulas

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$