Summary: To Exam 1 (Up to 2.8)

General Background: Chapter 1 and Appendices

There is a lot of algebra and trigonometry in Chapter 1, and Appendices A, B, C and D, so this is not an exhaustive list of everything you need to know, but there are some things we highlighted:

1. Construct the equation of a line (pt-slope form), Use the quadratic formula, find trigonometric values from right triangles/unit circle, function composition, simplify exponentials/logs using rules of exponents/logs.
2. Definitions: $|x|$, "one-to-one", "natural domain"
3. Be able to "Find the domain".
4. Use a **sign chart** to determine where an expression is positive/negative.
5. Know the difference between "inverse of a function" and the reciprocal of a function.
6. Given a formula for $f(x)$, be able to compute expressions like $f(2+h)$.

The Limit

1. Be able to compute limits algebraically and graphically.
2. Understand the meaning of, and be able to compute right and left-hand limits.
3. Work with and understand the definition of the limit: $\lim_{x\to a} f(x) = L$ means that we can keep the $f(x)$ values arbitrarily close to L by keeping the $x-$values sufficiently close to a.
4. Algebraic Methods to compute limits:
   (a) Simplify (e.g., absolute values)
   (b) Factor and Cancel
   (c) Multiply by Conjugate
   (d) Divide by x^n (Mainly for $x \to \infty$). Be careful! $x = \sqrt{x^2}$ if $x \geq 0$, but if $x < 0$, $x = -\sqrt{x^2}$
5. The Squeeze Theorem.
6. Horizontal/Vertical Asymptotes:
   (a) $x = a$ is a vertical asymptote for $f(x)$ if one of the following limits is infinite: $\displaystyle\lim_{x\to a^\pm} f(x)$
   (b) $y = b$ is a horizontal asymptote for $f(x)$ if one of the following is true: $\displaystyle\lim_{x\to\pm\infty} f(x) = b$

   Our template function: $\displaystyle\lim_{x\to\pm\infty} \frac{1}{x^r} = 0, \quad r > 0$

   (Note: x^r needs to be computable if $x \to -\infty$) Also, in a similar vein: $\displaystyle\lim_{x\to\infty} \mathrm{e}^{-x} = 0$ The inverse tangent has horizontal asymptotes:

   $$\lim_{x\to\pm\infty} \tan^{-1}(x) = \pm\frac{\pi}{2}$$

   And in general, if a function has a vertical asymptote at $x = a$, its inverse function will have a horizontal asymptote at $y = a$.
7. Intuition that can be used:
   (a) "$\infty + \infty = \infty$", but $\infty - \infty$ is not necessarily 0. (Similarly, the product but not the quotient)
   (b) If the denominator goes to zero, but the numerator does not, the limit is $\pm\infty$.

(c) If the denominator goes to $\pm\infty$, and the numerator does not, the overall limit goes to zero.

(d) Given a rational function, if the degree of the numerator is larger than the denominator, the function goes to $\pm\infty$ as $x \to \pm\infty$.

If the degree of the denominator is larger, then the function goes to zero (again, as $x \to \pm\infty$)

8. Limit Laws (Sect 1.3): Be able to summarize them (as on Quiz 3)

Continuity

1. Definition: f is continuous at $x = a$ if $\lim_{x\to a} f(x) = f(a)$
2. Interpretation of the definition: This means 3 things: (1) $f(a)$ exists, (2) The limit exists, and (3) Items 1 and 2 are the same number.
3. Show that a function is not continuous at a point by stating which of the three parts are violated.
4. Show that a function is continuous by using the definition.
5. Give the meaning of "continuous from the right" and "continuous from the left".
6. Theory about continuous functions: Know that our usual functions (see the list in Theorem 7) are all continuous *on their domain*. Know that the sum/difference, product/quotient of continuous functions is continuous (with a possibly restricted domain)
7. Be able to state and use the **Intermediate Value Theorem**:

   If f is continuous on $[a, b]$, and N is a number between $f(a)$ and $f(b)$, there is at least one c in $[a, b]$ so that $f(c) = N$.

   In practice, we usually use the IVT as:

   If f is continuous, and $f(x_1) > 0, f(x_2) < 0$, then there is a c between x_1 and x_2 where $f(c) = 0$ (f has at least one root in the interval between x_1 and x_2).

The Derivative

1. Know the definition of Average Velocity and the technique we use to get Instantaneous Velocity (aka Velocity)
2. Definition: $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$ Be able to compute this given numerical values of a, or as an arbitrary value of a (you would be given $f(x)$).
3. Interpretations of the Derivative of f at $x = a$:

   (a) The velocity at $x = a$.

   (b) The slope of the tangent line at $(a, f(a))$.

   (c) The instantaneous rate of change of f at $x = a$.
4. Equation of the Tangent Line at $x = a$: This is the line going through $(a, f(a))$ with slope $f'(a)$. The best (and fastest) way to write the line: $y - f(a) = f'(a)(x - a)$
5. Be able to compute the derivative, $f'(x)$ using the **definition**. (Remember, this means no shortcuts!)
6. Definition: A function f is differentiable at $x = a$ if $f'(a)$ exists. Understand what that means graphically.
7. (If covered) Understand the relationship between f being differentiable and f being continuous. That is, if f is differentiable at $x = a$, then it is automatically also continuous there. That doesn't work always in reverse- For example, $f(x) = |x|$ is continuous at $x = 0$, but $|x|$ is not differentiable at $x = 0$.
8. Sketching a derivative won't be on this exam, but it will appear on the next exam.