Exam 2 Review Questions

Sections 2.8-3.6 (and some 3.9? It depends on how far we get in class) will be on the exam... Also, don't forget to go over old quizzes and homework.

If you would like extra problems, the chapter reviews in our text are excellent. *Good Luck!*

1. Finish the definition:
   (a) The derivative of f is: $f'(x) =$
   (b) A function f is said to be *differentiable* at a point $x = a$ if:
   (c) A function f is said to be differentiable on the (open) interval (a, b) if:
2. Short Answer:
   (a) How do we define the inverse sine function? (Pay attention to the domain, range and whether the domain, range are angle measures or the ratios of a triangle).
   (b) What is a *normal line*?
   (c) How do we differentiate a function that involves the absolute value?
   (d) $\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta} =$? (You don't need to justify)
   (e) $\lim_{\theta\to 0} \frac{\cos(\theta)-1}{\theta} =$? (You don't need to justify)
   (f) $\lim_{\theta\to 0} \frac{\tan(3t)}{\sin(2t)} =$? (Do provide details)
   (g) If $f(x) = \sqrt{x}$, find a formula for $f'(x)$ *using the definition of the derivative.*
   (h) If $f(x) = 3/x$, find a formula for $f'(x)$ using the definition of the derivative.
3. Prove the Reciprocal Rule using the Product Rule (Hint: Start with $f(x) = 1/g(x)$, then write $f(x)g(x) = 1$).
4. Prove the Quotient Rule using the Product and Reciprocal Rules:
5. True or False, and explain:
   (a) The derivative of a polynomial is a polynomial.
   (b) If f is differentiable, then $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$
   (c) The derivative of $y = \sec^{-1}(x)$ is the derivative of $y = \cos(x)$.
   (d) $\frac{d}{dx}(10^x) = x10^{x-1}$
   (e) If $y = \ln|x|$, then $y' = \frac{1}{x}$
   (f) The equation of the tangent line to $y = x^2$ at $(1, 1)$ is: $y - 1 = 2x(x - 1)$
   (g) If $y = e^2$, then $y' = 2e$
   (h) If $y = |x^2 - x|$, then $y' = |2x - 1|$.
   (i) If $y = ax + b$, then $\frac{dy}{da} = x$
6. Find the equation of the tangent line to $x^3 + y^3 = 3xy$ at the point $(\frac{3}{2}, \frac{3}{2})$.
7. If $f(0) = 0$, and $f'(0) = 2$, find the derivative of $f(f(f(f(x))))$ at $x = 0$.
8. If $f(x) = 2x + \mathrm{e}^x$, find the equation of the tangent line **to the inverse** of f at $(1, 0)$. HINT: Do not try to compute f^{-1} algebraically.
9. Derive the formula for the derivative of $y = \csc^{-1}(x)$ using implicit differentiation.

10. Find the equation of the tangent line to $\sqrt{y} + xy^2 = 5$ at the point $(4, 1)$.

11. If $s^2t + t^3 = 1$, find $\frac{dt}{ds}$ and $\frac{ds}{dt}$. Are these expressions related to the derivative of a function and the derivative of its inverse?

12. If $y = x^3 - 2$ and $x = 3z^2 + 5$, then find $\frac{dy}{dz}$.

13. A space traveler is moving from left to right along the curve $y = x^2$. When she shuts off the engines, she will go off along the tangent line at that point. At what point should she shut off the engines in order to reach the point $(4, 15)$?

14. A particle moves in the plane according to the law $x = t^2 + 2t$, $y = 2t^3 - 6t$. Find the slope of the tangent line when $t = 0$. HINT: We can say that $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

15. Find h' in terms of f, g, f' and g', if: $h(x) = \dfrac{f(x)g(x)}{f(x) + g(x)}$

16. The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height.

    (a) Find the rate of change of the volume with respect to the radius if the height is constant.

    (b) Find the rate of change of the volume with respect to time if both the height and the radius are functions of time.

17. Find the coordinates of the point on the curve $y = (x - 2)^2$ at which the tangent line is perpendicular to the line $2x - y + 2 = 0$.

18. For what value(s) of A, B, C does the polynomial $y = Ax^2 + Bx + C$ satisfy the differential equation:

$$y'' + y' - 2y = x^2$$

    Hint: If $c_1x^2 + c_2x + c_3 = x^2$ for all x, then $c_1 = 1, c_2 = 0, c_3 = 0$.

19. If $V = \sin(w)$, $w = \sqrt{u}$, $u = t^2 + 3t$, compute: The rate of change of V with respect to w, the rate of change of V with respect to u, and the rate of change of V with respect to t.

20. Find all value(s) of k so that $y = e^{kt}$ satisfies the differential equation:

$$y'' - y' - 2y = 0$$

21. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

22. Differentiate. You may assume that y is a function of x, if not already defined explicitly. If you use implicit differentiation, solve for $\frac{dy}{dx}$.

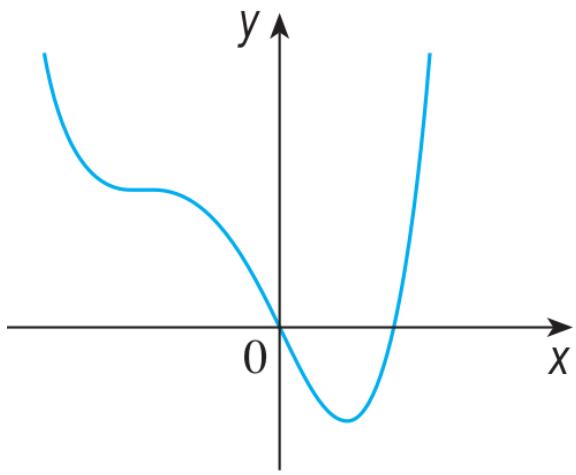
    (a) $y = \log_3(\sqrt{x} + 1)$
    (b) $\sqrt{2xy} + xy^3 = 5$
    (c) $y = \sqrt{x^2 + \sin(x)}$
    (d) $y = e^{\cos(x)} + \sin(5^x)$
    (e) $y = \cot(3x^2 + 5)$
    (f) $y = x^{\cos(x)}$
    (g) $y = \sqrt{\sin(\sqrt{x})}$
    (h) $\sqrt{x} + \sqrt[3]{y} = 1$
    (i) $x\tan(y) = y - 1$
    (j) $y = \sqrt{x}\, e^{x^2}(x^2+1)^{10}$ (Hint: Logarithmic Diff)
    (k) $y = \sin^{-1}\left(\tan^{-1}(x)\right)$
    (l) $y = \ln|\csc(3x) + \cot(3x)|$
    (m) $y = \frac{-2}{\sqrt[4]{t^3}}$
    (n) $y = x3^{-1/x}$
    (o) $y = x\tan^{-1}(\sqrt{x})$
    (p) $y = e^{2^{e^x}}$

(q) Let a be a positive constant. $y = x^a + a^x$

(r) $x^y = y^x$

(s) $y = \ln\left(\sqrt{\frac{3x+2}{3x-2}}\right)$

23. Given the graph of $f(x)$ below, next to it, sketch the graph of the derivative, $f'(x)$:



24. If f is the function whose graph is given below, let $h(x) = f(f(x))$, and use the graph below to estimate $h'(2)$.