Exam 2 Review: Sections 2.8-3.6 with some 3.9

This portion of the course covered the bulk of the formulas for differentiation, together with a few definitions and techniques. Remember that we also left 2.8 for this exam. I've left section 3.9 off the general review as it depends on how far we get in class- **Please make a note of the homework from this section for review**.

From 2.8, we should be able to plot the derivative given a graph of f, compute the derivative using the definition, and know the relationship between continuity and differentiability.

For Chapter 3, the following tables summarize the rules that we've had:

$f(x)$	$f'(x)$	Sect	$f(x)$	$f'(x)$	Sect
c	0	3.1	cf	cf'	3.1
x^n	nx^{n-1}	3.1	$f \pm g$	$f' \pm g'$	3.1
a^x	$a^x \ln(a)$	3.1	$f \cdot g$	$f'g + fg'$	3.2
e^x	e^x	3.1	$\frac{f}{g}$	$\frac{f'g-fg'}{g^2}$	3.2
$\log_a(x)$	$\frac{1}{x\ln(a)}$	3.6	$f(g(x))$	$f'(g(x))g'(x)$	3.4
$\ln(x)$	$\frac{1}{x}$	3.6	$f(x)^{g(x)}$	Logarithmic Diff	3.6
$\sin(x)$	$\cos(x)$	3.3	Eqn in x, y	Implicit Diff	3.5
$\cos(x)$	$-\sin(x)$	3.3			
$\tan(x)$	$\sec^2(x)$	3.3			
$\sec(x)$	$\sec(x)\tan(x)$	3.3			
$\csc(x)$	$-\csc(x)\cot(x)$	3.3			
$\cot(x)$	$-\csc^2(x)$	3.3			
$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$	3.5			
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$	3.5			

Vocabulary/Techniques:

- Be sure you distinguish between:

  a^x or $a^{f(x)}$ $\qquad$ x^a or $(f(x))^a$ $\qquad$ $f(x)^{g(x)}$

- Know the definition of "differentiable".
- Understand the relationship between "differentiable" and "continuous".
- Implicit Differentiation: A technique where we are given an equation with x, y. We treat y as a function of x, and differentiate without explicitly solving for y first.

  Example: $x^2y + \sqrt{xy} = 6x \to 2xy + x^2y' + \frac{1}{2}(xy)^{-\frac{1}{2}}(y + xy') = 6$

- Logarithmic Differentiation: A technique where we apply the logarithm to $y = f(x)$ before differentiating. Used for taking the derivative of complicated expressions, and needed for taking the derivative of $f(x)^{g(x)}$.

  Example: $y = x^x \to \ln(y) = x\ln(x) \to \frac{1}{y}y' = \ln(x) + 1 \to$ ... etc

- Differentiation of Inverses: If we know the derivative of $f(x)$, then we can determine the derivative of $f^{-1}(x)$. This technique was used to find derivatives of the inverse trig functions, for example:

  $y = f^{-1}(x) \Rightarrow f(y) = x \Rightarrow f'(y)y' = 1$ From this, we could write:

  $$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'(f^{-1}(x))}$$

  Alternatively, we say that if (a, b) is on the graph of f and $f'(a) = m$, then we know that (b, a) is on the graph of f^{-1}, and $\frac{df^{-1}}{dx}(b) = \frac{1}{m}$.

  NOTE: This is NOT the same as the derivative of $(f(x))^{-1} = \frac{1}{f(x)}$, which is

  $$\frac{d}{dx}\left((f(x))^{-1}\right) = -\left(f(x)\right)^{-2} f'(x) = \frac{-f'(x)}{(f(x))^2}$$

- We also have an alternative version of the Chain Rule that may be useful in certain cases.

  For example, if Volume is a function of Radius, Radius is a function of Pressure, Pressure is a function of time, then we can find the rate of change of Volume in terms of the Radius, or the Pressure, or the time. Respectively, this is:

  $$\frac{dV}{dR}, \qquad \frac{dV}{dP} = \frac{dV}{dR} \cdot \frac{dR}{dP}, \qquad \frac{dV}{dt} = \frac{dV}{dR} \cdot \frac{dR}{dP} \cdot \frac{dP}{dt}$$

  In fact, we could also find things like dR/dV, dP/dR, and so on because of the relationship between the derivative of a function and its inverse: $dx/dy = 1/(dy/dx)$.

- Things that come up in the inverse trig stuff: Be able to simplify expressions like $\tan(\cos^{-1}(x))$, $\sin(\tan^{-1}(x))$, etc. using an appropriate right triangle.

- Remember the logarithm rules:

  1. $A = e^{\ln(A)}$ for any $A > 0$.
  2. $\log(ab) = \log(a) + \log(b)$
  3. $\log(a/b) = \log(a) - \log(b)$
  4. $\log(a^b) = b\log(a)$

- Always simplify BEFORE differentiating. Example: To differentiate $y = x\sqrt{x}$, first rewrite as $y = x^{3/2}$