Extra Practice: 4.9, Ch 5

These problems are here to help you practice with just the last part of the course. See the sample exams for more general questions from the rest of the course.

Remember that formulas such as the following are found on pg. A37 (Appendix E)

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

1. For each of the following integrals, write the definition using the Riemann sum (and right endpoints), but do not evaluate them:

   (a) $\int_2^5 \sin(3x)\,dx$ (b) $\int_1^3 \sqrt{1+x}\,dx$ (c) $\int_0^2 e^x\,dx$

2. For each of the following integrals, write the definition using the Riemann sum, and then evaluate them (MUST use the limit of the Riemann sum for credit, and do not re-write them using the properties of the integral):

   (a) $\int_2^5 x^2\,dx$ (b) $\int_1^3 1 - 3x\,dx$ (c) $\int_0^5 1 + 2x^3\,dx$

3. For each of the following Riemann sums, evaluate the limit by first recognizing it as an appropriate integral. For part (a), see if you can write 4 equivalent integrals!

   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}\right)\sqrt{1+\frac{3i}{n}}$ (b) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + 3\cdot\frac{25i^2}{n^2}\right)\left(\frac{5}{n}\right)$ (c) $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(3+\frac{2i}{n}\right)\left(\frac{2}{n}\right)$

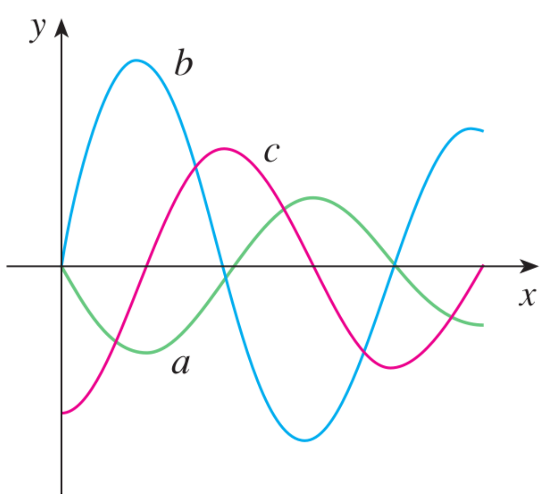
4. Suppose that

   $$\int_1^4 f(x)\,dx = 7 \qquad \int_2^4 f(x)\,dx = 5, \qquad \int_1^4 g(x)\,dx = 2$$

   Using only this information, compute the following:

   (a) $\int_1^4 4f(x)\,dx$ (b) $\int_1^4 8f(x) - 7g(x)\,dx$ (c) $\int_1^2 -f(x)\,dx$

5. The following figure shows the graphs of f, f' and $\int_0^x f(t)\,dt$. Identify each graph and explain your choices.



6. Find the derivative of the function:

(a) $F(x) = \int_0^x \frac{t^2}{1+t^2}\,dt$ (b) $F(x) = \int_x^1 \cos(t^2)\,dt$ (c) $F(x) = \int_{2x}^{3x+1} \frac{e^t}{t}\,dt$

7. Find the general indefinite integral:

(a) $\int \sqrt{x^3} + x^2 + \frac{1}{x}\,dx$ (b) $\int (u+4)(2u+1)\,du$ (c) $\int \sec^2(x) + \sin(x)\,dx$

8. Evaluate the integral:

(a) $\int_1^2 \left(\frac{x}{2} - \frac{2}{x}\right)\,dx$ (b) $\int_1^8 \frac{1+\sqrt[3]{u}}{\sqrt{u}}\,du$ (c) $\int_{-1}^2 x - 2|x|\,dx$ (d) $\int_0^1 \sqrt{1-x^2}\,dx$

9. Evaluate using the given substitution:

(a) $\int x^3(2+x^4)^5\,dx,\ u = 2+x^4$ (b) $\int \frac{dt}{(1-6t)^4},\ u = 1-6t$ (c) $\int \frac{\sec^2(1/x)}{x^2}\,dx,\ u = 1/x.$

10. Evaluate by finding a substitution:

(a) $\int \sin(4x)\,dx$ (b) $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$ (c) $\int \frac{z^2}{z^3+1}\,dz$ (d) $\int e^{2x}\,dx$