Derivative Warm-Ups

Differentiate:

- $y = \sec^2(\sin(x))$

$$y' = 2\sec(\sin(x)) \cdot \sec(\sin(x))\tan(\sin(x)) \cdot \cos(x)$$

- $y = 2^{1/x}$

$$y' = 2^{1/x}\ln(2) \cdot \left(-x^{-2}\right)$$

If $f(x) = x\mathrm{e}^x$, find $f'(x)$ and $f''(x)$.
SOLUTIONS:

$$f'(x) = \mathrm{e}^x + x\mathrm{e}^x$$

$$f''(x) = \mathrm{e}^x + \mathrm{e}^x + x\mathrm{e}^x = 2\mathrm{e}^x + x\mathrm{e}^x$$

Derivative Warm-Ups

1. $y = x^4 + 2x^3 + 6x$

$$y' = 4x^3 + 6x^2 + 6$$

2. $b(t) = \sqrt{t} + \frac{1}{3t^2}$

$$b(t) = t^{1/2} + \frac{1}{3}t^{-2} \quad \Rightarrow \quad b'(t) = \frac{1}{2}t^{-1/2} - \frac{2}{3}t^{-3}$$

3. $s(w) = \frac{w+2}{w+1}$

$$\frac{ds}{dw} = \frac{1 \cdot (w+1) - (w+2) \cdot 1}{(w+1)^2} = -\frac{1}{(w+1)^2}$$

Derivative Warm-Ups

1. $y(t) = tan^{-1}(3t)$

$$\frac{dy}{dt} = \frac{1}{1+(3t)^2} \cdot 3 = \frac{3}{1+9t^2}$$

2. $c(x) = \sqrt{\ln(x)}$

$$c'(x) = \frac{1}{2}(\ln(x))^{-1/2}\frac{1}{x}$$

3. $y = xf(x)$

$$y' = f(x) + xf'(x)$$

Given $\mathrm{e}^y = 2x^3y^2 + \mathrm{e}^2$, differentiate the expression twice (implicitly). Do NOT solve for y, y' or y''.
First derivative:

$$\mathrm{e}^y y' = (2x^3)'y^2 + 2x^3(y^2)' + (\mathrm{e}^2)'$$

$$\mathrm{e}^y y' = 6x^2y^2 + 2x^3 2yy' + 0$$

Second derivative

$$(\mathrm{e}^y)'y' + \mathrm{e}^y(y')' = (6x^2)'y^2 + 6x^2(y^2)' + (4x^3y)'y' + 4x^3y(y')'$$

$$\mathrm{e}^y y'y' + \mathrm{e}^y y'' = 12xy^2 + 12x^2yy' + (12x^2y + 4x^3y')y' + 4x^3yy''$$

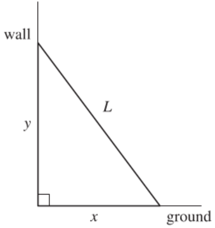
Differentiate: $y = \cos^x(x)$

$$\ln(y) = \ln(\cos^x(x)) = x\ln(\cos(x))$$

$$\frac{1}{y}y' = \ln(\cos(x)) + x\frac{1}{\cos(x)}(-\sin(x))$$

$$y' = \cos^x(x)\left(\ln(\cos(x)) - \frac{x\sin(x)}{\cos(x)}\right)$$

31. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?



$x^2 + y^2 = L^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

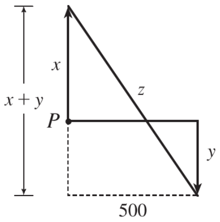
$2(3)(-0.15) + 2y(0.2) = 0$

Solve for y

$$y = 4$$

With $x = 3$ and $y = 4$, that forces $L = 5$.

17. A man starts walking north at 4 ft/s from a point P. Five minutes later a woman starts walking south at 5 ft/s from a point 500 ft due east of P. At what rate are the people moving apart 15 min after the woman starts walking?



$z^2 = (x+y)^2 + 500^2$

Known quantities:

$$\frac{dx}{dt} = 4 \quad \frac{dy}{dt} = 5 \text{ and } \begin{array}{c} 15 \text{ min} = 900 \text{ s} \\ 20 \text{ min} = 1200 \text{ s} \end{array}$$

Find $\frac{dz}{dt}$ after 15 min for woman.

$$2z\frac{dz}{dt} = 2(x+y)(x'+y')$$

We need x, y and then z, then we can solve for z'.

$$x = 4 \cdot 1200 = 4800, \quad y = 5 \cdot 900 = 4500, \quad z \approx 9313.4$$