M125
Fall 2019

Sample Final 2

Name:______________

Instructions: The exam is open book, open notes, and you may use anything from the class website, and you may use a calculator. You may not use any other sources of information. Remember- you are being tested over the ideas/techniques from Calculus; answers with no appropriate justification will receive no credit.

1. Short Answer:
   (a) True or False? $\dfrac{x^2-1}{x-1} = x+1$
   (b) If $f'(2)$ exists, then $\lim_{x\to 2} f(x) = f(2)$
   (c) If $f(x) = (2-3x)^{-1/2}$, find $f(0), f'(0)$ and $f''(0)$.
   (d) Show that the $x^4 + 4x + c = 0$ has at most one solution in the interval $[-1, 1]$.

2. Find dy/dx (solve for it if necessary):
   (a) $y = x\mathrm{e}^{g(\sqrt{x})}$ for some differentiable g.
   (b) $y = x^2 + 4^{1/x} + \sin^{-1}(3x+1) + \sec(x^2+x)$
   (c) $x\tan(y) = y - 1$
   (d) $\sqrt{x} + \sqrt{y} = 1$

3. Find $f'(x)$ directly from the definition of the derivative (using limits and without l'Hospital's rule): $f(x) = x^{-1}$

4. Find the limit if it exists. You may use any method (except for a numerical table).
   (a) $\lim_{x\to\infty} \sqrt{9x^2+x} - 3x$
   (b) $\lim_{x\to\pi^-} \dfrac{\sin(x)}{1-\cos(x)}$
   (c) $\lim_{x\to 0} \dfrac{x}{\tan^{-1}(4x)}$
   (d) $\lim_{x\to 1} x^{1/(1-x)}$

5. Differentiate: $F(x) = \displaystyle\int_{2x}^{x^2} \mathrm{e}^{t^2}\,dt$

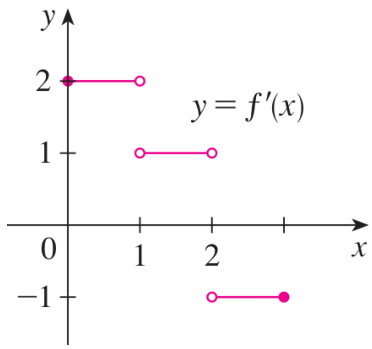
6. Evaluate the definite integral using the definition. To get credit, you must use the limit of the Riemann sum (use right endpoints and equal widths, as is our usual practice).
   $\displaystyle\int_2^5 x^2 + 1\,dx$

7. Evaluate the integral, if it exists
   (a) $\displaystyle\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\,du$
   (b) $\displaystyle\int 3^x + \frac{1}{x} + \sec^2(x)\,dx$
   (c) $\displaystyle\int_{\pi/4}^{\pi/4} \frac{t^4\tan(t)}{2+\cos(t)}\,dt$
   (d) $\displaystyle\int_0^3 |x^2-4|\,dx$

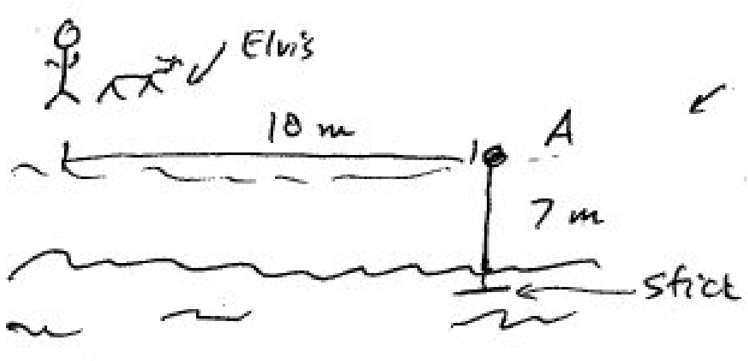
8. A water tank in the shape of an inverted cone with a circular base has a base radius of 2 meters and a height of 4 meters. If water is being pumped into the tank at a rate of 2 cubic meters per minute, find the rate at which the water level is rising when the water is 3 meters deep. ($V = \frac{1}{3}\pi r^2 h$)

9. Explain why the following is true (if it is): The function $f(x) = \sqrt{1+2x}$ can be well approximated by $(x+5)/3$ if x is approximately 4.

10. The graph of $f'(x)$ is shown in the figure. Sketch the graph of f if f is continuous and $f(0) = -1$.



11. You're standing with Elvis (the dog) on a straight shoreline, and you throw the stick in the water. Let us label as "A" the point on the shore closest to the stick, and suppose that distance is 7 meters. Suppose that the distance from you to the point A is 10 meters. Suppose that Elvis can run at 3 meters per second, and can swim at 2 meters per second. How far along the shore should Elvis run before going in to swim to the stick, if he wants to minimize the time it takes him to get to the stick?



12. Find m and b so that f is continuous and differentiable:

$$f(x) = \begin{cases} x^2 \text{ if } x \leq 2 \\ mx + b \text{ if } x > 2 \end{cases}$$

13. Find the absolute maximum and minimum of $f(x) = |x^2 - x|$ on the interval $[0, 2]$.