Exam 1 Review Questions

General notes: For multiple choice questions, no reasons are necessary. For free response questions, (1) Always give an algebraic reason for your answer (graphs are not sufficient), (2) You may not use shortcut rules for derivatives (these are covered in Chapter 3- If you don't know what I'm talking about, don't worry about it). (3) We will usually use the a, h version of the definition of the derivative- It's usually easier with the algebra.

1. Finish the definition:
   (a) $\lim_{x \to a} f(x) = L$ means:
   (b) The function f is continuous at $x = a$ if:
       Also, this definition implies three things:
   (c) The derivative of f at the point $x = a$ is:
   (d) A function f is said to be *differentiable* at a point $x = a$ if:

2. Find the domain: $f(x) = \sqrt{\dfrac{x^2 - 4}{1 - x^2}}$

3. Find the domain: $f(x) = \ln(x^2 + 2x - 3)$

4. The formula for the equation of the tangent line to $f(x)$ at $x = a$ is (use the point-slope form):

5. True or False, and give a short reason:
   (a) $\lim_{x \to 4} \left(\dfrac{2x}{x-4} - \dfrac{8}{x-4} \right) = \lim_{x \to 4} \left(\dfrac{2x}{x-4} \right) - \lim_{x \to 4} \left(\dfrac{8}{x-4} \right)$
   (b) $\lim_{x \to 1} \dfrac{x+4}{x^2-3} = \dfrac{\lim_{x \to 1}(x+4)}{\lim_{x \to 1}(x^2-3)}$
   (c) If f is continuous and $f(1) = 3$, $f(2) = 4$, then there is an r so that $f(r) = \pi$.
   (d) If $f'(1) = 3$, then $\lim_{x \to 1} f(x) - f(1) = 0$
   (e) All functions are continuous on their respective domains.
   (f) $\ln(x + 3) = \ln(x) + \ln(3)$
   (g) $\sin^{-1}(x) = \frac{1}{\sin(x)}$
   (h) A vertical line intersects the graph of a function at most once.
   (i) If, when taking a limit of a rational function, we get $\dfrac{0}{0}$, then the limit does not exist.

6. If the line $y = 2x - 1$ is tangent to the curve $y = f(x)$ at $x = -2$, then compute $f(-2)$ and $f'(-2)$

7. For the function $f(x) = x^2$, find and simplify the expression

$$\frac{f(x+h) - f(x-2h)}{3h}$$

8. Show that there must be at least one real solution to:

$$x^5 = x^2 + 4$$

9. Solve for x:

   (a) $x^2 < 2x + 8$

   (b) $\mathrm{e}^{x^2} = 4$

   (c) $\ln(5 - 2x) = -3$

   (d) $\ln(\ln(x)) = 1$

10. If $f(x) = 1 - x^3$, $g(x) = \frac{1}{x}$, compute the expression for $f \circ g$, $g \circ g$ (and simplify), $g \circ f$.

11. Compute each limit algebraically (if it exists):

   (a) $\lim\limits_{h \to 0} \dfrac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$

   (b) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{\frac{1}{x} - \frac{1}{2}}$

   (c) $\lim\limits_{x \to 2} \dfrac{x^2 - 2x - 3}{x - 3}$

   (d) $\lim\limits_{x \to \infty} (\sqrt{x^2 + 2x} - x)$

   (e) $\lim\limits_{x \to -\infty} \dfrac{3x + 2}{\sqrt{x^2 - 1}}$

   (f) $\lim\limits_{x \to 0} x^2 \sin\left(\dfrac{x^2 - 5x}{x^{26}}\right)$

   (g) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{2x^2 + 3x - 14}$

   (h) $\lim\limits_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$

12. Find all vertical and horizontal asymptotes for $\frac{2x+3}{\sqrt{x^2-2x-3}}$

13. Find all the values of a for which f will be continuous for all real values.

   (a) $f(x) = \begin{cases} 4 - x^2 & \text{if } x \leq 3 \\ 3 - ax & \text{if } x > 3 \end{cases}$

(b) $f(x) = \begin{cases} \frac{x^2-16}{x^2-4} & \text{if } x \neq \pm 2 \\ a & \text{if } x = \pm 2 \end{cases}$

14. The displacement (signed distance) of an object moving in a straight line is given by $s(t) = 1 + 2t + t^2/4$, where t is in seconds.

    (a) Find the average velocity over the time period $[1, 2]$.

    (b) Find the instantaneous velocity at $t = 1$.

15. Find the equation of the tangent line to $y = \frac{2}{1-3x}$ at $x = 0$.

16. For each function below, compute the derivative $f'(a)$ (use the definition of the derivative).

    (a) $f(x) = \sqrt{1+2x}$ at $a = 0$

    (b) $h(x) = x + \sqrt{x}$ at $a = 1$

    (c) $f(x) = \frac{2}{\sqrt{3-x}}$ at $a = -1$

17. If $f(x) = \sqrt{x}$, find a formula for $f'(x)$ *using the definition of the derivative.*

18. A space traveler is moving from left to right along the curve $y = x^2$. When she shuts off the engines, she will go off along the tangent line at that point. At what point should she shut off the engines in order to reach the point $(4, 15)$? (Hint: Label the unknown point on the graph of $y = x^2$ as (a, a^2)).

19. Some algebra and trig:

    (a) (App D) Solve for x: $|\tan(x)| = 1$

    (b) (App D) Find all x so that $\sin(x) \leq \frac{1}{2}$

    (c) (1.3) Express $F(x) = 1/\sqrt{x + \sqrt{x}}$ as the composition of three functions (you may not use the identity).

    (d) (1.6) Find the exact value: $\sin^{-1}(\sqrt{3}/2)$

    (e) (1.6) Simplify the expression: $\tan(\sin^{-1}(x))$

    (f) (1.6) Simplify the expression: $\sin(\cos^{-1}(x))$

    (g) (1.6) Solve for x: $1 - 2\ln(x) < 3$

    (h) (1.6) Solve for x: $e^{7-4x} = 6$

    (i) (1.6) Find f^{-1} and its domain if

    $$f(x) = \ln(e^x - 3)$$

    (j) (1.6) If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$ (Hint: Don't try to algebraically find f^{-1}, try to "eyeball" it)

(k) (1.6) If $f(x) = 1 + \sqrt{2 + 3x}$, find f^{-1} (algebraically).

20. More True or False, and give a short reason:

    (a) If $a < b$ (and $a > 0, b > 0$), then $\ln(a) < \ln(b)$.

    (b) If $f(s) = f(t)$, then $s = t$.

    (c) If f, g are functions, then $f \circ g = g \circ f$.

    (d) $\tan^{-1}(-1) = 3\pi/4$.

    (e) If x is any real number, then $x = \sqrt{x^2}$.

21. Finish the definition:

    (a) A horizontal asymptote is:

    (b) A vertical asymptote is:

    (c) A function f is one to one if:

    (d) A function f has even symmetry if:

    (e) A function f is decreasing if:

22. Graphical exercises: Please be sure you're comfortable with the graphical exercises given below.

    - Section 1.3, pg. 42: 3, 28.
    - Section 1.6, pg. 70: 5-8, 29-30
    - Section 2.2, pg. 96: 4-9.
    - Section 2.8, pg. 162: 3-11.
    - Ch 2 review, pg. 168: 42, 43, 47, 48