Summary: To Exam 1 (Up through 2.8)

General Background: Chapter 1 and Appendices

There is a lot of algebra and trigonometry in Chapter 1, and Appendices A, B, C and D, so this is not an exhaustive list of everything you need to know, but there are some things we highlighted:

1. Construct the equation of a line (pt-slope form), Use the quadratic formula, find trigonometric values from right triangles/unit circle, function composition, simplify exponentials/logs using rules of exponents/logs.
2. Definitions: $|x|$, "one-to-one", "natural domain"
3. Be able to "Find the domain".
4. Use a **sign chart** to determine where an expression is positive/negative.
5. Know the difference between "inverse of a function" and the reciprocal of a function.
6. Given a formula for $f(x)$, be able to compute expressions like $f(2+h)$.

The Limit: 2.1-2.3 and 2.6

1. Be able to compute limits algebraically and graphically.
2. Understand the meaning of, and be able to compute right and left-hand limits.
3. Work with and understand the definition of the limit

   $\lim_{x\to a} f(x) = L$ means that we can keep the $f(x)$ values arbitrarily close to L by keeping the $x-$values sufficiently close to a.
4. Algebraic Methods to compute limits:
   (a) Simplify (e.g., absolute values)
   (b) Factor and Cancel
   (c) Multiply by Conjugate
   (d) Divide by x^n (Mainly for $x \to \infty$). Be careful! $x = \sqrt{x^2}$ if $x \geq 0$, but if $x < 0$, $x = -\sqrt{x^2}$
5. The Squeeze Theorem.
6. Horizontal/Vertical Asymptotes:
   (a) $x = a$ is a vertical asymptote for $f(x)$ if one of the following:

$$\lim_{x\to a^{\pm}} f(x) = \pm\infty$$

(b) $y = b$ is a horizontal asymptote for $f(x)$ if one of the following:

$$\lim_{x\to\pm\infty} f(x) = b$$

Our template function: $\displaystyle\lim_{x\to\pm\infty} \frac{1}{x^r} = 0, \quad r > 0$
(Note: x^r needs to be computable if $x \to -\infty$) Also, in a similar vein: $\displaystyle\lim_{x\to\infty} \mathrm{e}^{-x} = 0$
The inverse tangent has horizontal asymptotes:

$$\lim_{x\to\pm\infty} \tan^{-1}(x) = \pm\frac{\pi}{2}$$

And in general, if a function has a vertical asymptote at $x = a$, its inverse function will have a horizontal asymptote at $y = a$.

7. Intuition that can be used:
   (a) "$\infty + \infty = \infty$", but $\infty - \infty$ is not necessarily 0. (Similarly, the product but not the quotient)
   (b) If the denominator goes to zero, but the numerator does not, the limit is $\pm\infty$.
   (c) If the denominator goes to $\pm\infty$, and the numerator does not, the overall limit goes to zero.
   (d) Given a rational function, if the degree of the numerator is larger than the denominator, the function goes to $\pm\infty$ as $x \to \pm\infty$.
   If the degree of the denominator is larger, then the function goes to zero (again, as $x \to \pm\infty$)
8. Limit Laws (2.3) You won't need to state them, but you may have to compute using them.

Continuity and IVT (2.5)

1. Definition: f is continuous at $x = a$ if $\lim_{x\to a} f(x) = f(a)$
2. Interpretation of the definition: This means 3 thing- What are they?
3. Show that a function is not continuous at a point by stating which of the three parts are violated.
4. Show that a function is continuous by using the definition.
5. Give the meaning of "continuous from the right" and "continuous from the left".
6. Theory about continuous functions: Know that our usual functions (see the list in Theorem 7) are all continuous *on their domain.* Know that the sum/difference, product/quotient of continuous functions is continuous (with a possibly restricted domain)

7. Be able to state and use the **Intermediate Value Theorem**:

   If f is continuous on $[a, b]$, and N is a number between $f(a)$ and $f(b)$, there is at least one c in $[a, b]$ so that $f(c) = N$.

   In practice, we usually use the IVT as:

   If f is continuous, and $f(x_1) > 0, f(x_2) < 0$, then there is a c between x_1 and x_2 where $f(c) = 0$ (f has at least one root in the interval between x_1 and x_2).

The Derivative (2.7-2.8)

1. Know the definition of average velocity and the technique we use to get instantaneous velocity (2.1)

2. Definition:
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$
   Be able to compute this given numerical values of a, or as an arbitrary value of a (you would be given $f(x)$).

3. Interpretations of the Derivative of f at $x = a$:
   (a) The velocity at $x = a$.
   (b) The slope of the tangent line at $(a, f(a))$.
   (c) The instantaneous rate of change of f at $x = a$.

4. Equation of the Tangent Line at $x = a$: This is the line going through $(a, f(a))$ with slope $f'(a)$. Use point-slope form.

5. Be able to compute the derivative $f'(x)$ using the definition (from 2.8).

6. Know what "differentiable" means.

7. Given the graph of f, be able to:
   (a) Sketch $f'(x)$
   (b) Say if f is differentiable at each $x = a$.
   (c) Understand the relationship between *continuity* and *differentiability*.

8. Be able to compute a higher derivative, like $f''(x)$ or $f'''(x)$.