Exam 2 Review Questions

The exam will cover sections 3.1-3.6 and 3.9, and be approximately one hour in length. You may use your textbook, notes, calculator, or anything on the class website. You may not use the internet for any other purpose. Remember, this is a timed exam, meaning that you may have time to look some things up, but not everything. Please study as if you will not have the book, then the time constraint will be easier to meet.

1. Short Answer:
   (a) How do we define the inverse sine function? (Pay attention to the domain, range and whether the domain, range are angle measures or the ratios of a triangle).
   (b) What is a *normal line*?
   (c) How do we differentiate a function that involves the absolute value?
   (d) $\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta}$ =? (You don't need to justify)
   (e) $\lim_{\theta\to 0}\frac{\cos(\theta)-1}{\theta}$ =? (You don't need to justify)
   (f) $\lim_{\theta\to 0}\frac{\tan(3t)}{\sin(2t)}$ =? (Do provide details)
2. Prove the "Reciprocal Rule" using the Product Rule (Hint: Start with $f(x) = 1/g(x)$, then write $f(x)g(x) = 1$).
3. Prove the Quotient Rule using the Product and Reciprocal Rules:
4. True or False, and explain (these are also available as a "quiz" on Canvas).
   (a) The derivative of a polynomial is a polynomial.
   (b) If f is differentiable, then $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$
   (c) The derivative of $y = \sec^{-1}(x)$ is the derivative of $y = \cos(x)$.
   (d) $\frac{d}{dx}(10^x) = x10^{x-1}$
   (e) If $y = \ln|x|$, then $y' = \frac{1}{x}$
   (f) The equation of the tangent line to $y = x^2$ at $(1,1)$ is: $y - 1 = 2x(x-1)$
   (g) If $y = e^2$, then $y' = 2e$
   (h) If $y = |x^2 - x|$, then $y' = |2x-1|$.
   (i) If $y = ax + b$, then $\frac{dy}{da} = x$
5. Find the equation of the tangent line to $x^3 + y^3 = 3xy$ at the point $(\frac{3}{2}, \frac{3}{2})$.
6. If $f(0) = 0$, and $f'(0) = 2$, find the derivative of $f(f(f(f(x))))$ at $x = 0$.
7. If $f(x) = 2x + \mathrm{e}^x$, find the equation of the tangent line **to the inverse** of f at $(1, 0)$. HINT: Do not try to compute f^{-1} algebraically.
8. Derive the formula for the derivative of $y = \csc^{-1}(x)$ using implicit differentiation.
9. Find the equation of the tangent line to $\sqrt{y} + xy^2 = 5$ at the point $(4, 1)$.
10. If $s^2t + t^3 = 1$, find $\frac{dt}{ds}$ and $\frac{ds}{dt}$. Are these expressions related to the derivative of a function and the derivative of its inverse?
11. If $y = x^3 - 2$ and $x = 3z^2 + 5$, then find $\frac{dy}{dz}$.

12. A space traveler is moving from left to right along the curve $y = x^2$. When she shuts off the engines, she will go off along the tangent line at that point. At what point should she shut off the engines in order to reach the point $(4, 15)$?

13. A particle moves in the plane according to the law $x = t^2 + 2t$, $y = 2t^3 - 6t$. Find the slope of the tangent line when $t = 0$. HINT: We can say that $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

14. Find h' in terms of f, g, f' and g', if: $h(x) = \dfrac{f(x)g(x)}{f(x) + g(x)}$

15. The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height.

    (a) Find the rate of change of the volume with respect to the radius if the height is constant.

    (b) Find the rate of change of the volume with respect to time if both the height and the radius are functions of time.

16. Find the coordinates of the point on the curve $y = (x-2)^2$ at which the tangent line is perpendicular to the line $2x - y + 2 = 0$.

17. For what value(s) of A, B, C does the polynomial $y = Ax^2 + Bx + C$ satisfy the differential equation:

$$y'' + y' - 2y = x^2$$

    Hint: If $c_1x^2 + c_2x + c_3 = x^2$ for all x, then $c_1 = 1, c_2 = 0, c_3 = 0$.

18. If $V = \sin(w)$, $w = \sqrt{u}$, $u = t^2 + 3t$, compute: The rate of change of V with respect to w, the rate of change of V with respect to u, and the rate of change of V with respect to t.

19. Find all value(s) of k so that $y = \mathrm{e}^{kt}$ satisfies the differential equation:

$$y'' - y' - 2y = 0$$

20. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1.

21. Differentiate. You may assume that y is a function of x, if not already defined explicitly. If you use implicit differentiation, solve for $\frac{dy}{dx}$.

    (a) $y = \log_3(\sqrt{x} + 1)$
    (b) $\sqrt{2xy} + xy^3 = 5$
    (c) $y = \sqrt{x^2 + \sin(x)}$
    (d) $y = \mathrm{e}^{\cos(x)} + \sin(5^x)$
    (e) $y = \cot(3x^2 + 5)$
    (f) $y = x^{\cos(x)}$
    (g) $y = \sqrt{\sin(\sqrt{x})}$
    (h) $\sqrt{x} + \sqrt[3]{y} = 1$
    (i) $x\tan(y) = y - 1$
    (j) $y = \sqrt{x}\,\mathrm{e}^{x^2}(x^2+1)^{10}$ (Hint: Logarithmic Diff)
    (k) $y = \sin^{-1}\left(\tan^{-1}(x)\right)$
    (l) $y = \ln|\csc(3x) + \cot(3x)|$
    (m) $y = \frac{-2}{\sqrt[4]{t^3}}$
    (n) $y = x3^{-1/x}$
    (o) $y = x\tan^{-1}(\sqrt{x})$
    (p) $y = \mathrm{e}^{2^{\mathrm{e}^x}}$
    (q) Let a be a positive constant. $y = x^a + a^x$
    (r) $x^y = y^x$
    (s) $y = \ln\left(\sqrt{\frac{3x+2}{3x-2}}\right)$

22. The related rates problems are all on a separate page.