Exam 3 Review Questions (Math 125)

1. True or False, and give a short reason:

   (a) If $f'(a) = 0$, then there is a local maximum or local minimum at $x = a$.

   (b) If f has a global minimum at $x = a$, then $f'(a) = 0$.

   (c) If $f''(2) = 0$, then $(2, f(2))$ is an inflection point for f.

   In the following, "increasing" or "decreasing" will mean for all real numbers x:

   (d) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x) + g(x)$ is increasing.

   (e) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x)g(x)$ is increasing.

   (f) If $f(x)$ is increasing, and $g(x)$ is decreasing, then $f(g(x))$ is decreasing.

2. Find the global maximum and minimum of the given function on the interval provided:

   (a) $f(x) = \sqrt{9 - x^2}$, $[-1, 2]$ (b) $g(x) = x - 2\cos(x)$, $[-\pi, \pi]$

3. Find the regions where f is increasing/decreasing: $f(x) = \frac{x}{(1+x)^2}$

4. For each function below, determine (i) where f is increasing/decreasing, (ii) where f is concave up/concave down, and (iii) find the local extrema.

   (a) $f(x) = x^3 - 12x + 2$

   (b) $f(x) = x\sqrt{6 - x}$

   (c) $f(x) = x - \sin(x)$, $0 < x < 4\pi$

5. Suppose $f(3) = 2$, $f'(3) = \frac{1}{2}$, and $f'(x) > 0$ and $f''(x) < 0$ for all x.

   (a) Sketch a possible graph for f.

   (b) How many roots does f have? (Explain):

   (c) Is it possible that $f'(2) = 1/3$? Why?

6. Let $f(x) = 2x + e^x$. Show that f has exactly one real root.

7. Suppose that $1 \le f'(x) \le 3$ for all $0 \le x \le 2$, and $f(0) = 1$. What is the largest and smallest that $f(2)$ can possibly be?

8. Linearize at $x = 0$: $y = \sqrt{x + 1}e^{-x^2}$. Use the linearization to estimate $\sqrt{\frac{3}{2}}e^{-\frac{1}{4}}$.

9. Estimate (by differentials) the change in the indicated quantity. Additionally, compute the exact change in the quantity.

(a) The period of oscillation, $T = 2\pi\sqrt{\frac{L}{32}}$, of a pendulum, if its length L is increased from 4 to 4.5.

(b) The velocity of air in the windpipe, $V = 16r - r^3$, if the radius of the windpipe, r, changes from 3 to 2 cm.

10. Let $f(x) = x^3 - 3x + 2$ on the interval $[-2, 2]$. Verify that the function satisfies all the hypotheses of the Mean Value Theorem, then find the values of c that satisfy its conclusion.

11. Find the limit, if it exists.

    (a) $\lim_{x\to 0} \frac{\sin^{-1}(x)}{x}$ (c) $\lim_{x\to 0^+} \sin(x)\ln(x)$ (e) $\lim_{x\to 0^+} x^{\sqrt{x}}$

    (b) $\lim_{x\to 0} \frac{x3^x}{3^x - 1}$ (d) $\lim_{x\to 0} \cot(2x)\sin(6x)$ (f) $\lim_{x\to 0^+} (4x+1)^{\cot(x)}$

12. Verify the given linear approximation (for small x).

    (a) $\sqrt[4]{1+2x} \approx 1 + \frac{1}{2}x$ (b) $\mathrm{e}^x \cos(x) \approx 1 + x$

13. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child paying out cord when 150 feet of cord is out? (Assume that the cord forms a line- actually an unrealistic assumption).

14. At 1:00 PM, a truck driver picked up a fare card at the entrance of a tollway. At 2:15 PM, the trucker pulled up to a toll booth 100 miles down the road. After computing the trucker's fare, the toll booth operator summoned a highway patrol officer who issued a speeding ticket to the trucker. (The speed limit on the tollway is 65 MPH).

    (a) The trucker claimed that she hadn't been speeding. Is this possible? Explain.

    (b) The fine for speeding is \$35.00 plus \$2.00 for each mph by which the speed limit is exceeded. What is the trucker's minimum fine?

15. Let $f(x) = \frac{1}{x}$

    (a) What does the Extreme Value Theorem (EVT) say about f on the interval $[0.1, 1]$?

    (b) Although f is continuous on $[1, \infty)$, it has no minimum value on this interval. Why doesn't this contradict the EVT?

16. Let f be a function so that $f(0) = 0$ and $\frac{1}{2} \leq f'(x) \leq 1$ for all x. Explain why $f(2)$ cannot be 3 (Hint: You might use a value theorem to help).

17. Find (if possible) the global maximum and minimum of $g(x) = x/(x^3 + 2)$ on $[0, \infty)$.

18. A poster is to contain 50 square inches of printed matter, with 4 inch margins at the top and bottom, and 2 inch margins on each side. What dimensions for the poster would use the least amount of paper?

19. Henry, who is in a rowboat 2 miles from the nearest point B on a straight shoreline, notices smoke billowing from his house, which is 6 miles down the shoreline from point B. He figures he can row at 6 miles per hour and run at 10 miles per hour. How should he proceed in order to get to the house in the least amount of time?

20. Show that the rectangle with the maximum perimeter that can be inscribed in a circle is a square.

21. Find the points on the hyperbola $x^2/4 - y^2 = 1$ that are closest to the point $(5, 0)$.

22. Find the equation of a line through $(3, 5)$ that cuts off the least area from the first quadrant.

23. A woman is in the water 1 mile from the closest point B on the straight shoreline. She wants to get to a town 3 miles down from B, so she can swim part of the way, and walk part of the way. If she can swim at 2 miles per hour and walk at 4 miles per hour, where on the shore should she land to minimize the time to town?

24. Find the value of the indicated sum by first expanding the sum:

    (a) $\displaystyle\sum_{k=1}^{7} \cos(k\pi)$ (b) $\displaystyle\sum_{j=2}^{6} (j+1)^2$

25. Write the indicated sum in sigma notation: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots - \frac{1}{50}$

26. Find the value of the sum by using the formulas on pg A37 (Theorem 3).

    (a) $\displaystyle\sum_{k=1}^{7} [(k-1)(4k+3)]$ (b) $\displaystyle\sum_{n=1}^{10} 5n^2(n+4)$

27. Find the most general antiderivative of $f(x) = 3\cos(x) - 6\sin(x)$.

28. If $f'(x) = (x+1)(x-2)$, find the most general $f(x)$.

29. Find the most general $f(x)$, if $f''(x) = 12x + 24x^2$.

30. Find f, if $f'(t) = 2t - 3\sin(t)$, and $f(0) = 5$.

31. **Graphical Exercises** Please look these problems over as well- They include some graphical analysis. Sect 4.2: 7, Sect 4.3: 5-6, 7-8, 31-32, Sect 4.9: 51-55