Exam 3: Applications of the Derivative

In this section of the course, we've looked at 3.10, then 4.1-4.4 then 4.7, 4.9 and Appendix E. Below is a short summary of the material.

1. (3.10) Linear approximations and differentials.

$$\begin{aligned} L(x) &= f(a) + f'(a)(x-a) \\ \Delta y &= f(x+\Delta x) - f(x) \\ dy &= f'(x)\,dx \end{aligned}$$

   The first formula is the linearization of f. The second is the **actual** change in y as x changes from x to $x + \Delta x$. The third equation is the formula for the differential dy which is used to approximate Δy. In this section, we learn that $L(x)$ can be used to approximate $f(x)$ close to $x = a$, and similarly, that dy can approximate Δy. Graphically show $\Delta x, \Delta y$ and dy.

2. (4.1) Maximums and Minimums (Absolute or Global)

   In this section, we looked at how to find the absolute (or global) maximum and minimum values of a function on a closed interval.

   - The **Extreme Value Theorem** told us when to expect a global max/min.
   - To find the global max/min for f on a closed interval:
     - Compute the critical points for f.
     - Build a table using the critical points and endpoints for f.
     - The largest value in the table is the max, smallest is the min.

3. (4.2) The Mean Value Theorem

   In this section, we actually have two important theorems- Rolle's theorem and the Mean Value Theorem. You can remember Rolle's theorem by using the Mean Value Theorem and taking $f(a) = f(b) = 0$.

   Be sure you can state the MVT. Be able to "compute the c value from the MVT", Be able to show that an equation has at most n solutions. Given a bound on $f'(x)$, and $f(a)$, state how large or how small $f(b)$ is.

4. (4.3) The shapes of graphs (Local extrema)

   (a) Use the derivatives and/or graph of the derivatives to determine where f is inc, dec, CU, CD.
   (b) Define Critical Points (CPs) and Inflection Points.
   (c) Understand the relationship between CPs and local extrema (Fermat's Theorem).
   (d) The first derivative test.
   (e) The second derivative test.
   (f) The closed interval method for finding absolute extrema.

5. (4.4) l'Hospital's rule

   l'Hospital's rule gives us an extra technique for computing limits. The basic theorem concludes with:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

   Please know when this rule can be applied. We then looked at other forms that can will require some algebra before applying the rule. For example, the limit of a product $f(x)g(x)$ or the limit of an exponentiation, $f(x)^{g(x)}$.

6. (4.7) Optimization.

   Several goals here: Be able to convert the "story problem" into mathematical statements, then solve. In these problems, we're typically going to try to find the extreme points of a function, typically of more than one variable, with some side condition. The side condition allows us to substitute values into the function, making the function depend on only one variable, and from there we have a regular extremum problem.

7. (4.9) Antiderivatives.

   We looked at the concept (and definition) of the antiderivative, and constructed a table (much like our table of derivatives). We construct antiderivatives algebraically and graphically (given $f'(x)$, construct the graph of $f(x)$). Given acceleration, be able to compute the velocity and position functions.

8. Appendix E: Sigma notation and sums.

   We defined sigma notation- Be able to write a sum using sigma notation, and be able to expand a sum given sigma notation. Be able to use the formulas for the sums in Theorem 3 to compute the sums in certain situations (like exercises 21-35). Be able to evaluate a telescoping sum (be writing it out and looking to see what terms cancel).