Exam 3 Review solutions

1. True or False, and give a short reason:

   (a) If $f'(a) = 0$, then there is a local maximum or local minimum at $x = a$.
   FALSE: For example, $f(x) = x^3$ at $x = 0$.

   (b) If f has a global minimum at $x = a$, then $f'(a) = 0$.
   FALSE. It could be that $x = a$ is an endpoint for an interval, or a point where $f'(a)$ does not exist.

   (c) If $f''(2) = 0$, then $(2, f(2))$ is an inflection point for f.
   FALSE. An inflection point is one where the concavity changes.

   In the following, "increasing" or "decreasing" will mean for all real numbers x:

   (d) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x) + g(x)$ is increasing.
   TRUE: Since f, g are increasing, f', g' are both positive so $f' + g'$ is positive as well.

   (e) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x)g(x)$ is increasing.
   FALSE: It might be true, but
   $$(fg)' = f'g + fg'$$
   so we might have that $g < 0$ and $f < 0$, for example. In that case, the derivative would be negative.

   (f) If $f(x)$ is increasing, and $g(x)$ is decreasing, then $f(g(x))$ is decreasing.
   TRUE:
   $$(f(g(x)))' = f'(g(x))g'(x)$$
   so f' (evaluated at $g(x)$) is positive, and g' is positive.

2. Find the global maximum and minimum of the given function on the interval provided:

   (a) $f(x) = \sqrt{9 - x^2}$, $[-1, 2]$
   SOLUTION: $f'(x) = -x/\sqrt{9 - x^2}$, so we add the critical point $x = 0$. Build a table:

x	-1	0	2
y	2.82	3.0	2.23

   So the global minimum is $y = 2.23$ and the global maximum is $y = 3.0$

   (b) $g(x) = x - 2\cos(x)$, $[-\pi, \pi]$
   SOLUTION: $g'(x) = 1+2\sin(x)$. Use a triangle and/or unit circle to find the values of $x = -5\pi/6$ and $x = -\pi/6$. Now the table:

x	$-\pi$	$-5\pi/6$	$-\pi/6$	π
y	-1.14	-0.88	-2.25	5.14

   The global minimium is approx -2.25 and the global max is approx 5.14.

3. Find the regions where f is increasing/decreasing: $f(x) = \frac{x}{(1+x)^2}$
   SOLUTION: Simplifying the derivative, we get:
   $$f'(x) = \frac{1 - x}{(1 + x)^3}$$
   Sign chart (include $x = -1$ although it is a vertical asymptote):

$f'(x)$	$-$	$+$	$-$
	$x < -1$	$-1 < x < 1$	$x > 1$

   Therefore, f is decreasing if $x < -1$ and if $x > 1$, and increasing if $-1 < x < 1$.

4. For each function below, determine (i) where f is increasing/decreasing, (ii) where f is concave up/concave down, and (iii) find the local extrema.

   (a) $f(x) = x^3 - 12x + 2$ (See Exercise 33, 4.3)
   Hint: $f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x-2)(x+2)$, then look at a sign chart, and $f''(x) = 6x$.

   (b) $f(x) = x\sqrt{6-x}$ (See Exercise 39, 4.3)
   Hints: The domain is $x \le 6$, and we can simplify

$$f'(x) = \frac{3}{2} \cdot \frac{x-4}{\sqrt{6-x}}$$

$$f''(x) = \frac{3}{4} \cdot \frac{x-8}{(6-x)^{3/2}}$$

   (c) $f(x) = x - \sin(x)$, $0 < x < 4\pi$ (See Exercise 44. 4.3)
   SOLUTION: $f'(x) = 1 - \cos(x)$, and $\cos(x) = 1$ at $x = 0, 2\pi, 4\pi$. Since we do not include $0, 4\pi$ in the interval, we have:

$f'(x)$	$+$	$+$
	$0 < x < 2\pi$	$2\pi < x < 4\pi$

   Therefore, $f'(x) > 0$, and f is always increasing.
   For concavity, $f''(x) = \sin(x)$. The sine is posiive (so f is increasing) on $(0, \pi)$ and $(2\pi, 3\pi)$. The sine is negative (so f is decreasing) on $(\pi, 2\pi)$ and $(3\pi, 4\pi)$.
   For local extrema, the only critical point is 2π, and from our analysis of the first derivative, we see that this is a plateau, not a local extreme point (so there are no local extrema).

5. Suppose $f(3) = 2$, $f'(3) = \frac{1}{2}$, and $f'(x) > 0$ and $f''(x) < 0$ for all x.

   (a) Sketch a possible graph for f.
   SOLUTION: At the point $(3, 2)$, the function increases and is concave down.

   (b) How many roots does f have? (Explain):
   SOLUTION: The function f can have at most 1 root. If it had two roots, Rolle's theorem would mean that $f'(x) = 0$ for some x between the roots- But we're told that $f'(x) > 0$.

   (c) Is it possible that $f'(2) = 1/3$? Why?
   SOLUTION: Since $f'(3) = 1/2$ and $1/3 < 1/2$, then this implies that f' is increasing. However, that implies that $f'' > 0$, but $f''(x) < 0$ for all x. Therefore, the given value of $f'(x)$ is not possible.

6. Let $f(x) = 2x + e^x$. Show that f has exactly one real root.

   - We see that $f(-1) = -2 + e^{-1} < 0$ and $f(0) = 1 > 0$. Therefore, by the IVT there is at least one real root.
   - The derivative is $f'(x) = 2 + e^x$, which is always positive (since e^x is always greater than 0). Therefore, by Rolle's Theorem, there is at most 1 real root.
   - By the previous 2 items, there is exactly one (real) root.

7. Suppose that $1 \le f'(x) \le 3$ for all $0 \le x \le 2$, and $f(0) = 1$. What is the largest and smallest that $f(2)$ can possibly be?

   SOLUTION: We can use the MVT:

$$\frac{f(2) - f(0)}{2 - 0} = f'(c) \quad \Rightarrow \quad 1 \le \frac{f(2) - 1}{2} \le 3 \quad \Rightarrow \quad 3 \le f(2) \le 7$$

8. Linearize at $x = 0$:

$$y = \sqrt{x+1}\,\mathrm{e}^{-x^2}$$

Use the linearization to estimate $\sqrt{\frac{3}{2}}\mathrm{e}^{-\frac{1}{4}}$

SOLUTION: At $x = 0$, we have $y = \sqrt{1}\mathrm{e}^0 = 1$. Now for the slope:

$$f'(x) = \frac{1}{2}(x+1)^{-1/2}\mathrm{e}^{-x^2} + \sqrt{x+1}\,\mathrm{e}^{-x^2}(-2x)$$

so $f'(0) = \frac{1}{2}$, and $L(x) = 1 + \frac{1}{2}x$. Therefore, (note that the question asks you to approximate $f(1/2)$):

$$f(1/2) \approx L(1/2) = 1 + \frac{1}{2}\cdot\frac{1}{2} = \frac{5}{4}$$

9. Estimate using differentials the change in the quantity:

   (a) The period of oscillation:

   $$T(L) = \frac{2\pi}{\sqrt{32}}L^{1/2} \quad \Rightarrow \quad dT = \frac{2\pi}{4\sqrt{2}}\,\frac{1}{2}L^{-1/2}\,dL = \frac{2\pi}{4\sqrt{2}}\,\frac{1}{2}\,\frac{1}{\sqrt{4}}\,\frac{1}{2} = \frac{\pi}{16\sqrt{2}} \approx 0.1388$$

   (b) The velocity of air in the windpipe when the radius changes from 3 to 2.

   $$V(r) = 16r - r^3 \quad \Rightarrow \quad dV = (16 - 3r^2)\,dr = (16 - 3\cdot 9)\cdot(-1) = 11$$

   (Note: This approximation is terrible because the change in r is so large).

10. Let $f(x) = x^3 - 3x + 2$ on the interval $[-2, 2]$. Verify that the function satisfies all the hypotheses of the Mean Value Theorem, then find the values of c that satisfy its conclusion.

    SOLUTION: f is a polynomial, so it is continuous and differentiable at all real numbers. Now we find c so that

    $$f'(c) = \frac{f(2) - f(-2)}{2 - (-2)} = \frac{4 - 0}{4} = 1$$

    And $f'(c) = 3c^2 - 3$, so:

    $$3c^2 - 3 = 1 \quad \Rightarrow \quad c = \pm\frac{2}{\sqrt{3}}$$

    Both of these are in the interval $[-2, 2]$.

11. Find the limit, if it exists.

    (a) $\displaystyle\lim_{x\to 0}\frac{\sin^{-1}(x)}{x} = \lim_{x\to 0}\frac{1}{\sqrt{1-x^2}} = 1$

    (b) $\displaystyle\lim_{x\to 0}\frac{x3^x}{3^x - 1} = \lim_{x\to 0}\frac{3^x + x3^x\ln(3)}{3^x\ln(3)} = \frac{1}{\ln(3)}$

    (c) $\displaystyle\lim_{x\to 0^+}\sin(x)\ln(x) = \lim_{x\to 0^+}\frac{\ln(x)}{1/\sin(x)} = \lim_{x\to 0^+}\frac{\ln(x)}{\csc(x)} = \lim_{x\to 0^+}\frac{1/x}{-\csc(x)\cot(x)} =$

    To finish this off, we rewrite the denominator, and we can express this as:

    $$\lim_{x\to 0^+}\frac{1}{x}\cdot(-\sin(x)\tan(x)) = \lim_{x\to 0^+}\frac{-\sin(x)}{x}\cdot\tan(x) = 1\cdot 0 = 0$$

    (d) $\displaystyle\lim_{x\to 0}\cot(2x)\sin(6x)$ We'll use the limit of $\sin(\theta)/\theta$:

    $$= \lim_{x\to 0}\frac{\cos(2x)}{1}\cdot\frac{1}{\sin(2x)}\cdot\frac{\sin(6x)}{1} = \lim_{x\to 0}\frac{\cos(2x)}{1}\cdot\frac{2x}{\sin(2x)}\cdot\frac{\sin(6x)}{6x}\,\frac{6x}{2x} = 3$$

(e) $\lim_{x\to 0^+} x^{\sqrt{x}}$

First, take the log (at the end, we exponentiate):

$$\lim_{x\to 0^+} \ln\left(x^{\sqrt{x}}\right) = \lim_{x\to 0^+} \sqrt{x}\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{x^{-1/2}}$$

Use l'Hospital's rule and simplify:

$$\lim_{x\to 0^+} \frac{1/x}{-(1/2)x^{-3/2}} = \lim_{x\to 0^+} \frac{1}{x}\cdot 2x^{3/2} = \lim_{)} x \to 0^+ - 2\sqrt{x} = 0$$

Now exponentiate, so that overall the limit is $e^0 = 1$.

(f) $\lim_{x\to 0^+} (4x+1)^{\cot(x)}$

Same idea as before- first take logs, then some algebra to get the right form for l'Hospital's rule.

$$\lim_{x\to 0^+} \cot(x)\ln(4x+1) = \lim_{x\to 0^+} \frac{\ln(4x+1)}{\tan(x)} = \lim_{x\to 0^+} \frac{\frac{4}{4x+1}}{\sec^2(x)} = \lim_{x\to 0^+} \frac{4\cos^2(x)}{4x+1} = 4$$

12. Verify the given linear approximation (for small x).

(a) $\sqrt[4]{1+2x} \approx 1 + \frac{1}{2}x$

SOLUTION: If $f(x) = \sqrt[4]{1+2x}$, we should find the tangent line approximation for $f(x)$ at $x = 0$ (from the statement "for small x") is the given line. Let's see- we need a point and a slope. The point will be $(0, 1)$, and the slope is

$$f'(x) = \frac{1}{4}(1+2x)^{-3/4}\cdot 2 \quad \Rightarrow \quad f'(0) = \frac{1}{2}$$

The tangent line is given by $y - 1 = \frac{1}{2}(x-0)$, or $y = 1 + \frac{1}{2}x$

(b) $e^x\cos(x) \approx 1 + x$

SOLUTION: If $f(x) = e^x\cos(x)$, we should find the tangent line approximation for $f(x)$ at $x = 0$ (from the statement "for small x") is the given line. Let's see- we need a point and a slope. The point will be $(0, 1)$, and the slope is

$$f'(x) = e^x\cos(x) - e^x\sin(x) \quad \Rightarrow \quad f'(0) = 1$$

The tangent line is given by $y - 1 = 1(x-0)$, or $y = 1 + x$

13. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child paying out cord when 150 feet of cord is out? (Assume that the cord forms a line- actually an unrealistic assumption).

Note: As with all word problems, your notation may be different than mine First, draw a picture of a right triangle, with height 90, hypotenuse $y(t)$, the other leg $x(t)$. Note that these are varying until after we differentiate. So, we have that

$$(y(t))^2 = (x(t))^2 + 90^2$$

and

$$2y(t)\frac{dy}{dt} = 2x(t)\frac{dx}{dt}$$

We want to find $\frac{dy}{dt}$ when $y = 150$ and $\frac{dx}{dt} = 5$. We need to know $x(t)$, so use the first equation:

$$150^2 - 90^2 = x^2 \Rightarrow x = 120$$

so

$$2\cdot 150\frac{dy}{dt} = 2\cdot 120\cdot 5 \Rightarrow \frac{dy}{dt} = 4$$

14. At 1:00 PM, a truck driver picked up a fare card at the entrance of a tollway. At 2:15 PM, the trucker pulled up to a toll booth 100 miles down the road. After computing the trucker's fare, the toll booth operator summoned a highway patrol officer who issued a speeding ticket to the trucker. (The speed limit on the tollway is 65 MPH).

    (a) The trucker claimed that she hadn't been speeding. Is this possible? Explain.
    SOLUTION: Nope. Not possible. The trucker went 100 miles in 1.25 hours, which is not possible if you go (at a maximum) of 65 miles per hour (which would only get you (at a max) 81.25 miles). In terms of the MVT:
    $$\frac{\text{Change in Position}}{\text{Change in time}} = \frac{100}{1.25} = 80$$
    So we can guarantee that at some point in time, the trucker's speedometer read exactly 80 MPH.

    (b) The fine for speeding is \$35.00 plus \$2.00 for each mph by which the speed limit is exceeded. What is the trucker's minimum fine? By the last computation, the trucker had an *average* speed of 80 mph, so we can guarantee (by the MVT) that at some point, the speedometer read exactly 80. So, this gives \$35.00 + \$2.00 (15) = \$65.00

15. Let $f(x) = \frac{1}{x}$

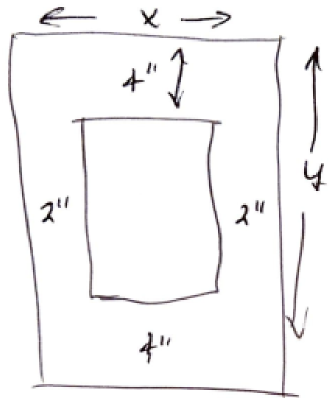
    (a) What does the Extreme Value Theorem (EVT) say about f on the interval $[0.1, 1]$?
    SOLUTION: Since f is continuous on this closed interval, there is a global max and global min (on the interval).

    (b) Although f is continuous on $[1, \infty)$, it has no minimum value on this interval. Why doesn't this contradict the EVT?
    SOLUTION: The EVT was stated on an interval of the form $[a, b]$, which implies that we cannot allow a, b to be infinite.

16. Let f be a function so that $f(0) = 0$ and $\frac{1}{2} \le f'(x) \le 1$ for all x. Explain why $f(2)$ cannot be 3 (Hint: You might use a value theorem to help).
    SOLUTION: We know that:
    $$\frac{f(2) - f(0)}{2 - 0} = f'(c)$$
    for some c in $(0, 2)$. Using the restrictions on the derivative,
    $$\frac{1}{2} \le \frac{f(2)}{2} \le 1$$
    so that $1 \le f(2) \le 2$.

17. Find (if possible) the global maximum and minimum of $g(x) = x/(x^3 + 2)$ on $[0, \infty)$.
    SOLUTION: First, get the critical points.
    $$g'(x) = \frac{1 \cdot (x^3 + 2) - x(3x^2)}{(x^3 + 2)^2} = 0 \quad \Rightarrow \quad x = 1$$
    Check the sign of the derivative to see if it changes on the interval. We see that if $0 \le x < 1$, then $g'(x) > 0$, and if $x > 1$, then $g'(x) < 0$. Therefore, the global maximum is when $x = 1$, $g(1) = 1/3$. There is no global minimum for $x > 1$, as $g(x) \to 0$ as $x \to \infty$. The function does go through $(0, 0)$ and increases to $1/3$, so 0 is the global minimum.

18. A poster is to contain 50 square inches of printed matter, with 4 inch margins at the top and bottom, and 2 inch margins on each side. What dimensions for the poster would use the least amount of paper?



SOLUTION: Let x, y be the width and height of the paper. Then the printed matter has dimensions $x - 4$ by $y - 8$. We then want to minimize the area of the paper, given that the printed matter is 50 square inches:

$$\min \; A = xy \qquad \text{such that } (x-4)(y-8) = 50$$

We want A to depend only on x, so solve the other equation for y:

$$y = \frac{50}{x-4} + 8 \quad \Rightarrow \quad A = \frac{50x}{x-4} + 8x, \quad 0 < x < \infty$$

Now we find the critical points for A:

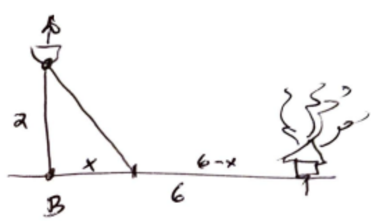
$$\frac{dA}{dx} = \frac{50(x-4) - 50x}{(x-4)^2} = \frac{8x^2 - 64x - 72}{(x-4)^2} = \frac{8(x+1)(x-9)}{(x-4)^2}$$

so $x = -1$ and $x = 9$. We only consider $x = 9$ since $x > 0$, and look to see if this we have a max or min here. A quick sign chart will help us:

$x+1$	$+$	$+$
$x-9$	$-$	$+$
$(x-4)^2$	$+$	$+$
	$0 < x < 9$	$x > 9$

Therefore, the function is decreasing until $x = 9$, then increases afterward so we have a global minimum at $x = 9$, and $y = \frac{50}{9-4} + 8 = 18$.

19. Henry, who is in a rowboat 2 miles from the nearest point B on a straight shoreline, notices smoke billowing from his house, which is 6 miles down the shoreline from point B. He figures he can row at 6 miles per hour and run at 10 miles per hour. How should he proceed in order to get to the house in the least amount of time?



SOLUTION: See the figure. Recall that distance equals rate times time, so $t = d/r$. If we let x be the point on the shore where we land, then there are two distances- the one on the water, which is $\sqrt{x^2+4}$, and the distance on land, $6 - x$.

$$T = \frac{\sqrt{x^2+4}}{2} + \frac{6-x}{10}, \qquad 0 \le x \le 6$$

Now we compute the critical point(s). After differentiation, put the result as a single fraction:

$$T'(x) = \frac{10x - 6\sqrt{x^2+4}}{60\sqrt{x^2+4}} = 0 \quad \Rightarrow \quad 10x = 6\sqrt{x^2+4}$$

Square both sides to solve for x:

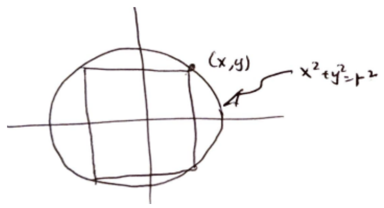
$$100x^2 = 36(x^2+4) \quad \Rightarrow \quad 64x^2 = 144 \quad \Rightarrow \quad x = \frac{12}{8} = \frac{3}{2}$$

Now check the time values at the endpoints and critical points:

$$T(0) = 0.93, \quad T(3/2) = 0.87, \quad T(6) = 1.05$$

The shortest time will be using $x = 3/2$.

20. Show that the rectangle with the maximum perimeter that can be inscribed in a circle is a square.



SOLUTION: Given the rectangle shown with side lengths $2x \times 2y$, we have:

$$\max P = 4x + 4y, \qquad \text{such that } x^2 + y^2 = r^2$$

Make P a function of one variable,

$$P = 4x + 4\sqrt{r^2 - x^2}, \quad 0 \le x \le r$$

Now critical points:

$$P' = 4 + 4\frac{1}{2}(r^2 - x^2)^{-1/2}(-2x) = 4 - \frac{4x}{\sqrt{r^2 - x^2}}$$

Set this to zero (you'll need to square both sides):

$$\frac{4x}{\sqrt{r^2 - x^2}} = 4 \quad \Rightarrow \quad 4x = 4\sqrt{r^2 - x^2} \quad \Rightarrow \quad 16x^2 = 16(r^2 - x^2) \quad \Rightarrow \quad 32x^2 = 16r^2 \quad \Rightarrow \quad x = \frac{r}{\sqrt{2}}$$

And $y = r/\sqrt{2}$, which makes the shape a perfect square.

21. Find the points on the hyperbola $x^2/4 - y^2 = 1$ that are closest to the point $(5, 0)$.

SOLUTION: One note that will make the problem easier. Recall that the distance formula is

$$d = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

The minimum of this value actually occurs at the same point (x, y) as the square of the distance,

$$D = (x - x_0)^2 + (y - y_0)^2$$

And so we can minimize the square of the distance rather than the distance itself (it makes the derivatives much easier). With that note, we have the distance (squared) between a point (x, y) and the point $(5, 0)$, but the (x, y) must be on the hyperbola:

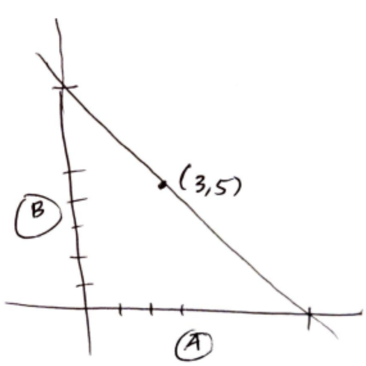
$$\min (x - 5)^2 + (y - 0)^2 \qquad \text{such that } \frac{x^2}{4} - y^2 = 1$$

We'll go ahead and solve for y in the second expression and substitute it into the first:

$$f(x) = (x - 5)^2 + \frac{x^2}{4} - 1 = \frac{5}{4}x^2 - 10x + 24 \quad \Rightarrow \quad \frac{5}{2}x - 10 = 0 \quad \Rightarrow \quad x = 4$$

This is where the global minimum occurs for f since f is a parabola (opening upwards). Further, we also need the $y-$coordinate, which is $\sqrt{3}$.

22. Find the equation of a line through $(3, 5)$ that cuts off the least area from the first quadrant.



SOLUTION: See the graph. Given a line, $y = mx + b$, then the length marked A is found by setting $y = 0$, and solve for x, so length $A = -b/m$. Similarly, the length marked B is found by substituting $x = 0$, so length $B = b$. So, for the line $y = mx + b$, the triangle has area

$$A = \frac{1}{2} b \frac{-b}{m}$$

We need to be sure that $(3, 5)$ is on the line, so $5 = 3m + b$, or $m = (5 - b)/3$. Substituting this into A, we get a function of one variable:

$$A = \frac{1}{2} b \cdot \frac{-b}{1} \frac{3}{5 - b} = \frac{3b^2}{b - 5}$$

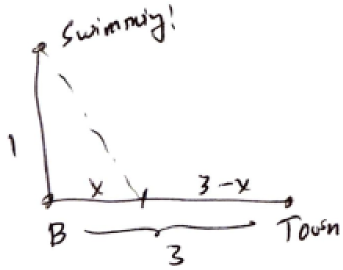
Are there restrictions on b? We need $m < 0$, so that $b > 5$. Now differentiating as usual, we get

$$\frac{dA}{db} = \frac{3}{2}\frac{b(b-10)}{(b-5)^2} = 0 \quad \Rightarrow \quad b = 0, b = 10$$

We only keep $b = 10$, and from the first derivative test (for $b > 5$), we see that A is always decreasing until $b = 10$, then always increasing afterward, so we have a global minimum at $b = 10$. This gives $m = -5/3$, and our line is

$$y = -\frac{5}{3}x + 10$$

23. A woman is in the water 1 mile from the closest point B on the straight shoreline. She wants to get to a town 3 miles down from B, so she can swim part of the way, and walk part of the way. If she can swim at 2 miles per hour and walk at 4 miles per hour, where on the shore should she land to minimize the time to town?



SOLUTION: See the figure. This one is set up just like the other one above. In this case,

$$T(x) = \frac{\sqrt{1+x^2}}{2} + \frac{3-x}{4} \quad \Rightarrow \quad T'(x) = 0 \quad \Rightarrow \quad x = \frac{1}{\sqrt{3}}$$

I left off the details of computing the derivative- Very similar to the previous problem. Now we evaluate T at $x = 0, x = 3$ and $x = 1/\sqrt{3}$, and we have a minimum when $x = 1/\sqrt{3}$.

24. Find the value of the indicated sum by first expanding the sum:

   (a) $\sum_{k=1}^{7} \cos(k\pi) = \cos(\pi) + \cos(2\pi) + \cos(3\pi) + \cos(4\pi) + \cos(5\pi) + \cos(6\pi) + \cos(7\pi) = -1$

   (b) $\sum_{j=2}^{6} (j+1)^2 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = 135$

25. Write the indicated sum in sigma notation: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots - \frac{1}{50} = \sum_{n=1}^{50} (-1)^{n+1}\frac{1}{n}$

26. Find the value of the sum by using the formulas on pg A37 (Theorem 3).

   (a) $\sum_{k=1}^{7} [(k-1)(4k+3)] =$ Expanding, we get

$$\sum_{k=1}^{7} 4k^2 - k - 3 = 4\sum_{k=1}^{7} k^2 - \sum_{k=1}^{7} k - 3\sum_{k=1}^{7} 1 = 4\frac{7\cdot 8\cdot 15}{6} - \frac{7\cdot 8}{2} - 3\cdot 7 = 511$$

   (b) $\sum_{n=1}^{10} 5n^2(n+4)$ Similar to the last problem,

$$5\sum_{n=1}^{10} n^3 + 20\sum_{n=1}^{10} n^2 = 5\cdot\left(\frac{10\cdot 11}{2}\right)^2 + 20\cdot\frac{10\cdot 11\cdot 21}{6} = 22825$$

27. Find the most general antiderivative of $f(x) = 3\cos(x) - 6\sin(x)$.

    SOLUTION: $F(x) = 3\sin(x) + 6\cos(x) + C$

28. If $f'(x) = (x+1)(x-2)$, find the most general $f(x)$.

    SOLUTION: Rewrite the derivative as $f'(x) = x^2 - x - 2$, so that $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + C$

29. Find the most general $f(x)$, if $f''(x) = 12x + 24x^2$.

    SOLUTION: $f'(x) = 6x^2 + 8x^3 + C_1$, and $f(x) = 2x^3 + 2x^4 + C_1x + C_2$

30. Find f, if $f'(t) = 2t - 3\sin(t)$, and $f(0) = 5$.

$$f(t) = t^2 + 3\cos(t) + C \qquad 5 = 0^2 + 3 \cdot 1 + C \quad \Rightarrow \quad C = 2$$

    Therefore, $f(t) = t^2 + 3\cos(t) + 2$.

31. **Graphical Exercises** Please look these problems over as well- They include some graphical analysis. Sect 4.2: 7, Sect 4.3: 5-6, 7-8, 31-32, Sect 4.9: 51-55