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Algebraically Determining Limits- Limit Laws

The limit laws state that, if

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$$\lim_{x\to a} p(x) = p(a)$$

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$$\lim_{x \to 3} \frac{x^2 - 1}{x + 1} = \frac{3^2 - 1}{3 + 1} = \frac{8}{4} = 2$$

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Algebraic Technique: Simplify and Cancel if possible!

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$$\lim_{h \to 0} \frac{9 + 6h + h^2 - 9}{h} = \lim_{h \to 0} \frac{6h + h^2}{h} = \lim_{h \to 0} \frac{h(6+h)}{h} =$$

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$$\lim_{t \to 0} \frac{(t^2+9)-9}{t^2(\sqrt{t^2+9}+3)} = \lim_{t \to 0} \frac{1}{\sqrt{t^2+9}+3} = \frac{1}{6}$$

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Squeeze theorem: On the board.