

I certify that the work on this exam is my own and that I have not discussed the specific contents of this exam with anyone prior to taking it.

Initials:

Instructions: You may not use any notes, the text or your colleagues (please keep your eyes on your own work!). Remember- you are being tested over the ideas/techniques from Calculus; answers with no appropriate justification will receive no credit. There are some formulas at the end of the exam you might find helpful.

1. Give the definition of the derivative: $f'(x) =$
2. State the Fundamental Theorem of Calculus. It begins by having f be a continuous function on $[a, b]$.
3. True or False (and give a short reason):
 - (a) If f is continuous at $x = a$, then f is differentiable at $x = a$.
 - (b) If $3 \leq f(x) \leq 5$ for all x , then $6 \leq \int_1^3 f(x) dx \leq 10$.
 - (c) All continuous functions have antiderivatives.
 - (d) $\int_{-2}^1 -x^{-2} dx = x^{-1} \Big|_{-2}^1 = \frac{3}{2}$
4. Find $f'(x)$ directly from the definition of the derivative $f(x) = \sqrt{1+x}$.
5. Derive the formula for the derivative of $y = \sin^{-1}(x)$.
6. Find dy/dx (solve for it, if necessary):
 - (a) $y = \sin^3(x^2 + 1) + \tan^{-1}(x)$
 - (b) $y = 3^{1/x} + \sec(x)$
 - (c) $\sqrt{x+y} = 4xy$
7. Find the limit, if it exists (you may use any technique from class):
 - (a) $\lim_{x \rightarrow 0} \frac{1 - e^{-2x}}{\sec(x)}$
 - (b) $\lim_{x \rightarrow 4^+} \frac{x-4}{|x-4|}$
 - (c) $\lim_{x \rightarrow -\infty} \sqrt{\frac{2x^2-1}{x+8x^2}}$
 - (d) $\lim_{x \rightarrow 0^+} x^x$
8. Evaluate the Riemann sum by first writing it as an appropriate definite integral: $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \sqrt{1 + \frac{3i}{n}}$.
9. Differentiate: $F(x) = \int_{\sqrt{x}}^{x^2} \frac{t}{1+t} dt$
10. Evaluate the definite integral by using the definition. That is, first write the Riemann sum, then take the appropriate limit. (You must use the Riemann sum to get credit)

$$\int_0^3 1 + 3x dx$$

11. Evaluate, or find the general indefinite integral.

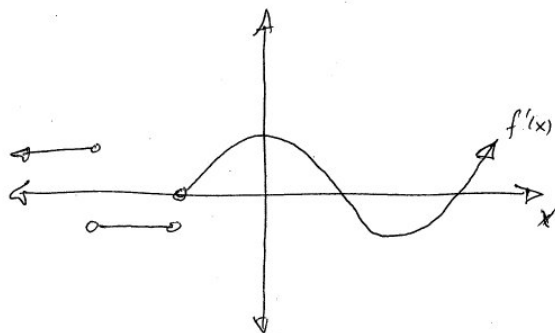
(a) $\int \sqrt{x^3} + \frac{1}{x^2 + 1} dx$ (b) $\int_{-1}^1 t(1-t) dt$ (c) $\int_0^1 5x - 5^x dx$

12. Evaluate:

(a) $\int_0^1 \frac{d}{dx} (e^{\tan^{-1}(x)}) dx$ (b) $\frac{d}{dx} \int_0^1 e^{\tan^{-1}(x)} dx$ (c) $\frac{d}{dx} \int_0^x e^{\tan^{-1}(t)} dt$

13. Given the graph of the derivative, $f'(x)$, below, answer the following questions:

- (a) Find all intervals on which f is increasing.
- (b) Find all intervals on which f is concave up.
- (c) Sketch a possible graph of f if we require that $f(0) = -1$.



14. A rectangle is to be inscribed between the x -axis and the upper part of the graph of $y = 8 - x^2$ (symmetric about the y -axis). For example, one such rectangle might have vertices: $(1, 0)$, $(1, 7)$, $(-1, 7)$, $(-1, 0)$ which would have an area of 14. Find the dimensions of the rectangle that will give the largest area.

15. Find all values of c and d so that f is continuous at all real numbers:

$$f(x) = \begin{cases} 2x^2 - 1 & \text{if } x < 0 \\ cx + d & \text{if } 0 \leq x \leq 1 \\ \sqrt{x+3} & \text{if } x > 1 \end{cases}$$

Be sure it is clear from your work that you understand the definition of continuity.

Formulas

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2} \right)^2$$