Exam 1 Review Questions

Please also review the old quizzes, and be sure that you understand the homework problems. Remember that I am looking to see if you understand the underlying issues, not just that you can come up with an answer. That means that answers with no justification may be marked off (on the other hand, I will try to give reasonable partial credit as well).

1. Three key definitions (finish them up):
   (a) $\lim_{x\to a} f(x) = L$ means (in words):
   (b) The function f is continuous at $x = a$ if (short answer) .
   This definition implies three things:
   (c) The derivative of f at the point $x = a$ is (short answer):

2. Find the domain: (a) $f(x) = \sqrt{\dfrac{x^2-4}{1-x^2}}$ (b) $f(x) = \ln(x^2 + 2x - 3)$

3. The formula for the equation of the tangent line to $f(x)$ at $x = a$ is (use the point-slope form):

4. True or False, and give a short reason:
   (a) $\lim_{x\to 4}\left(\dfrac{2x}{x-4} - \dfrac{8}{x-4}\right) = \lim_{x\to 4}\left(\dfrac{2x}{x-4}\right) - \lim_{x\to 4}\left(\dfrac{8}{x-4}\right)$
   (b) $\lim_{x\to 1} \dfrac{x+4}{x^2-3} = \dfrac{\lim_{x\to 1}(x+4)}{\lim_{x\to 1}(x^2-3)}$
   (c) If f is continuous and $f(1) = 3$, $f(2) = 4$, then there is an r so that $f(r) = \pi$.
   (d) All functions are continuous on their respective domains.
   (e) $\ln(x + 3) = \ln(x) + \ln(3)$
   (f) $\sin^{-1}(x) = \frac{1}{\sin(x)}$
   (g) We use the Squeeze Theorem to show $\lim_{x\to 0} x^2 \cos(1/x)$ is zero.
   (h) A vertical line intersects the graph of a function at most once.
   (i) If, when taking a limit of a rational function, we get $\dfrac{0}{0}$, then the limit DNE.
   (j) If x is any real number, then $x = \sqrt{x^2}$.

5. If the line $y = 2x - 1$ is tangent to the curve $y = f(x)$ at $x = -2$, then compute $f(-2)$ and $f'(-2)$

6. For the function $f(x) = x^2$, find and simplify the expression $\dfrac{f(x+h) - f(x-2h)}{3h}$

7. Show that there must be at least one real solution to: $x^5 = x^2 + 4$.

8. Solve for x:

(a) $x^2 < 2x + 8$ (b) $e^{x^2} = 4$ (c) $\ln(5 - 2x) = -3$

9. If $f(x) = 1 - x^3$, $g(x) = \frac{1}{x}$, compute the expression for $f \circ g$, $g \circ g$ (and simplify), $g \circ f$.

10. Compute each limit algebraically (if it exists):

(a) $\lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$ (d) $\lim_{x\to\infty} (\sqrt{x^2 + 2x} - x)$ (g) $\lim_{x\to 2} \frac{x^2 - 4}{2x^2 + 3x - 14}$

(b) $\lim_{x\to 2} \frac{x^2 - 4}{\frac{1}{x} - \frac{1}{2}}$ (e) $\lim_{x\to -\infty} \frac{3x + 2}{\sqrt{x^2 - 1}}$ (h) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{|x|}\right)$

(c) $\lim_{x\to 2} \frac{x^2 - 2x - 3}{x - 3}$ (f) $\lim_{x\to 0} x^2 \sin\left(\frac{x^2 - 5x}{x^{26}}\right)$ (i) $\lim_{v\to 4^-} \frac{4 - v}{|4 - v|}$

11. Find all vertical and horizontal asymptotes for $\frac{2x+3}{\sqrt{x^2-2x-3}}$

12. Find all the values of a for which f will be continuous for all real values.

(a) $f(x) = \begin{cases} 4 - x^2 & \text{if } x \le 3 \\ 3 - ax & \text{if } x > 3 \end{cases}$ (b) $f(x) = \begin{cases} x^2 - 2 & \text{if } x \le a \\ 2x - 1 & \text{if } x > a \end{cases}$ (c) $f(x) = \begin{cases} \frac{x^2-16}{x^2-4} & \text{if } x \neq \pm 2 \\ a & \text{if } x = \pm 2 \end{cases}$

13. The displacement (signed distance) of an object moving in a straight line is given by $s(t) = 1 + 2t + t^2/4$, where t is in seconds.

(a) Find the average velocity over the time period $[1, 2]$.

(b) Find the instantaneous velocity at $t = 1$.

14. Find the equation of the tangent line to $y = \frac{2}{1-3x}$ at $x = 0$.

15. For each function below, compute the derivative $f'(a)$ (use the definition of the derivative).

(a) $f(x) = \sqrt{1 + 2x}$ (c) $h(x) = x + \sqrt{x}$ (e) $f(x) = \frac{x}{x^2-1}$

(b) $g(x) = \frac{1}{x^2}$ (d) $f(x) = \frac{2}{\sqrt{3-x}}$

16. A space traveler is moving from left to right along the curve $y = x^2$. When she shuts off the engines, she will go off along the tangent line at that point. At what point should she shut off the engines in order to reach the point $(4, 15)$? (Hint: Label the unknown point on the graph of $y = x^2$ as (a, a^2)).

17. Some algebra and trig:

(a) Solve for x: $|\tan(x)| = 1$

(b) Find all x so that $\sin(x) \le \frac{1}{2}$

(c) Express $F(x) = 1/\sqrt{x + \sqrt{x}}$ as the composition of three functions (you may not use the identity).

(d) Find the exact value: $\sin^{-1}(\sqrt{3}/2)$

(e) Simplify the expression: $\tan(\sin^{-1}(x))$

(f) Simplify the expression: $\sin(\cos^{-1}(x))$

(g) Solve for x: $1 - 2\ln(x) < 3$

(h) Solve for x: $e^{7-4x} = 6$

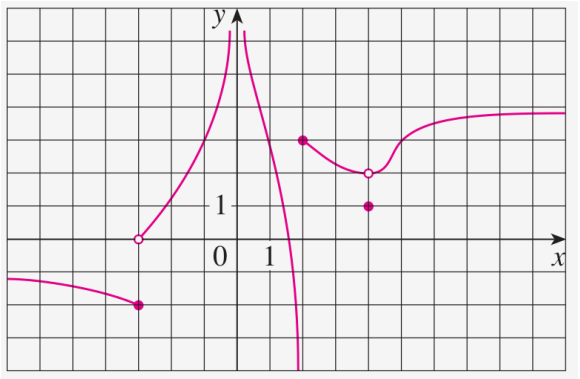
(i) Find f^{-1} and its domain if

$$f(x) = \ln(e^x - 3)$$

(j) If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$ (Hint: Don't try to algebraically find f^{-1}, try to "eyeball" it)

(k) If $f(x) = 1 + \sqrt{2 + 3x}$, find f^{-1} (algebraically).

18. Find each limit, or explain why it does not exist.



(a) $\lim_{x \to 2^+} f(x)$

(b) $\lim_{x \to -3^+} f(x)$

(c) $\lim_{x \to -3} f(x)$

(d) $\lim_{x \to 4} f(x)$

(e) $\lim_{x \to 0} f(x)$

(f) $\lim_{x \to 2^-} f(x)$

(g) $\lim_{x \to \infty} f(x)$

(h) $\lim_{x \to -\infty} f(x)$

19. The graph of f is shown. State, with reasons, the numbers at which f is not differentiable.