Exam 2 Review: Topics from 3.1-3.6 with 3.9

This portion of the course covered the bulk of the formulas for differentiation, together with a few definitions and techniques.

For Chapter 3, the following tables summarize the rules that we've had:

$f(x)$	$f'(x)$	Sect
c	0	3.1
x^n	nx^{n-1}	3.1
a^x	$a^x \ln(a)$	3.1
e^x	e^x	3.1
$\log_a(x)$	$\frac{1}{x\ln(a)}$	3.6
$\ln(x)$	$\frac{1}{x}$	3.6
$\sin(x)$	$\cos(x)$	3.3
$\cos(x)$	$-\sin(x)$	3.3
$\tan(x)$	$\sec^2(x)$	3.3
$\sec(x)$	$\sec(x)\tan(x)$	3.3
$\csc(x)$	$-\csc(x)\cot(x)$	3.3
$\cot(x)$	$-\csc^2(x)$	3.3
$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$	3.5
$\tan^{-1}(x)$	$\frac{1}{1+x^2}$	3.5

$f(x)$	$f'(x)$	Sect
cf	cf'	3.1
$f \pm g$	$f' \pm g'$	3.1
$f \cdot g$	$f'g + fg'$	3.2
$\frac{f}{g}$	$\frac{f'g - fg'}{g^2}$	3.2
$f(g(x))$	$f'(g(x))g'(x)$	3.4
$f(x)^{g(x)}$	Logarithmic Diff	3.6
Eqn in x, y	Implicit Diff	3.5

Vocabulary/Techniques:

- Be sure you distinguish between:

$$a^x \text{ or } a^{f(x)} \qquad x^a \text{ or } (f(x))^a \qquad f(x)^{g(x)}$$

- Know the definition of "differentiable".
- Understand the relationship between "differentiable" and "continuous".
- Implicit Differentiation: A technique where we are given an equation with x, y. We treat y as a function of x, and differentiate without explicitly solving for y first.

  Example: $x^2y + \sqrt{xy} = 6x \to 2xy + x^2y' + \frac{1}{2}(xy)^{-\frac{1}{2}}(y + xy') = 6$
- Logarithmic Differentiation: A technique where we apply the logarithm to $y = f(x)$ before differentiating. Used for taking the derivative of complicated expressions, and needed for taking the derivative of $f(x)^{g(x)}$.

  Example: $y = x^x \to \ln(y) = x\ln(x) \to \frac{1}{y}y' = \ln(x) + 1 \to \ldots$ etc
- Differentiation of Inverses: If we know the derivative of $f(x)$, then we can determine the derivative of $f^{-1}(x)$. This technique was used to find derivatives of the inverse trig functions, for example:

  $y = f^{-1}(x) \Rightarrow f(y) = x \Rightarrow f'(y)y' = 1$ From this, we could write:

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'(f^{-1}(x))}$$

Alternatively, we say that if (a, b) is on the graph of f and $f'(a) = m$, then we know that (b, a) is on the graph of f^{-1}, and $\frac{df^{-1}}{dx}(b) = \frac{1}{m}$.

NOTE: This is NOT the same as the derivative of $(f(x))^{-1} = \frac{1}{f(x)}$, which is

$$\frac{d}{dx}\left((f(x))^{-1}\right) = -\left(f(x)\right)^{-2} f'(x) = \frac{-f'(x)}{(f(x))^2}$$

- We also have an alternative version of the Chain Rule that may be useful in certain cases.

  For example, if Volume is a function of Radius, Radius is a function of Pressure, Pressure is a function of time, then we can find the rate of change of Volume in terms of the Radius, or the Pressure, or the time. Respectively, this is:

  $$\frac{dV}{dR}, \qquad \frac{dV}{dP} = \frac{dV}{dR} \cdot \frac{dR}{dP}, \qquad \frac{dV}{dt} = \frac{dV}{dR} \cdot \frac{dR}{dP} \cdot \frac{dP}{dt}$$

  In fact, we could also find things like dR/dV, dP/dR, and so on because of the relationship between the derivative of a function and its inverse: $dx/dy = 1/(dy/dx)$.

- Things that come up in the inverse trig stuff: Be able to simplify expressions like $\tan(\cos^{-1}(x))$, $\sin(\tan^{-1}(x))$, etc. using an appropriate right triangle.

- Remember the logarithm rules:
  1. $A = \mathrm{e}^{\ln(A)}$ for any $A > 0$.
  2. $\log(ab) = \log(a) + \log(b)$
  3. $\log(a/b) = \log(a) - \log(b)$
  4. $\log(a^b) = b\log(a)$

- Always simplify BEFORE differentiating. Example: To differentiate $y = x\sqrt{x}$, first rewrite as $y = x^{3/2}$

- For the related rates (Section 3.9), the workflow through the story problems will typically go as:
  - Draw a picture
  - Label constants and variables that are needed (careful that you don't include too many).
  - Determine the functional relationship
    * Geometric formula for a circle, square or triangle.
    * Geometric formula for a sphere or cone (these would be provided).
    * Similar triangles.

  Think of this list as a recommendation, but some problems may rely on things we haven't listed.