Exam 3: Applications of the Derivative

In this section of the course, we've looked at 3.10, then 4.1-4.4 and 4.7. Below is a short summary of the material. The details are not included- be sure to have your textbook handy so that you can look things up that you are not familiar with.

1. (3.10) Linear approximations and differentials.

   To summarize, we use the tangent line to approximate the function at a given point, $x = a$ (this is $L(x)$ below). We can also use dy (the differential) to approximate Δy.

   $$\begin{aligned} L(x) &= f(a) + f'(a)(x-a) \\ \Delta y &= f(x+\Delta x) - f(x) \\ dy &= f'(x)\,dx \end{aligned}$$

   The first formula is the linearization of f. The second is the **actual** change in y as x changes from x to $x + \Delta x$. The third equation is the formula for the differential dy which is used to approximate Δy.

2. (4.1) Maximums and Minimums (Absolute or Global)

   In this section, we looked at how to find the absolute (or global) maximum and minimum values of a function on a closed interval.

   - The **Extreme Value Theorem** told us when to expect a global max/min.
   - To find the global max/min for f on a closed interval:
     - Compute the critical points for f.
     - Build a table using the critical points and endpoints for f.
     - The largest value in the table is the max, smallest is the min.

3. (4.2) The Mean Value Theorem

   In this section, we actually have two important theorems- Rolle's theorem and the Mean Value Theorem. You can remember Rolle's theorem by using the Mean Value Theorem and taking $f(a) = f(b) = 0$.

   Be sure you can state the MVT- the best way to recall it is to remember that the average velocity is being set equal to the instantaneous velocity:

   $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

   Be able to "compute the c value from the MVT", Be able to show that an equation has at most n solutions. Given a bound on $f'(x)$, and $f(a)$, state how large or how small $f(b)$ is.

4. (4.3) The shapes of graphs (Local extrema)

(a) Use the derivatives and/or graph of the derivatives to determine where f is inc, dec, CU, CD.

(b) Define Critical Points (CPs) and Inflection Points.

(c) Understand the relationship between CPs and local extrema (Fermat's Theorem).

(d) The first derivative test (for local extrema)

(e) The second derivative test (for local extrema)

(f) The closed interval method for finding absolute extrema.

5. (4.4) l'Hospital's rule

   l'Hospital's rule gives us an extra technique for computing limits. The basic theorem concludes with:

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}.$$

   provided this limit exists or is $\pm\infty$. Please know when this rule can be applied. We then looked at other forms that can will require some algebra before applying the rule.

   Key Algebraic Techniques: Convert functions of the form $f(x)g(x)$ or $f(x)^{g(x)}$ into a form suitable for l'Hospital's rule.

6. (4.7) Optimization.

   In "optimization", there are three basic tasks:

   - Convert the story problem into mathematical statements.
   - If the function we are optimizing has more than one variable, there should be side conditions that can be used to substitute into the function (to make it a function of one variable).
   - Once you have a function of one variable, be sure to consider its domain- We may need to use the "closed interval method" for example.