Exam 3 Review Questions

This is not meant to be an all exhaustive list, rather it is meant to give you some random questions out of context. It is important that you understand the homework and the quizzes as well as these exercises.

1. Short Answer:
   (a) Give the definition of a **critical point** for a function f:
   (b) State the three "Value Theorems" (don't just name them, but also state each):
   (c) What is the procedure for finding the maximum or minimum of a function $y = f(x)$ on a closed interval, $[a, b]$.
   (d) How do we determine if a function has a local maximum or minimum?
   (e) What is meant by *linearizing* a function?

2. True or False, and give a short reason:
   (a) If $f'(a) = 0$, then there is a local maximum or local minimum at $x = a$.
   (b) There is a vertical asymptote at $x = 2$ for $\frac{\sqrt{x^2+5}-3}{x^2-2x}$
   (c) If f has a global minimum at $x = a$, then $f'(a) = 0$.
   (d) If $f''(2) = 0$, then $(2, f(2))$ is an inflection point for f.

   In the following, "increasing" or "decreasing" will mean for all real numbers x:
   (e) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x) + g(x)$ is increasing.
   (f) If $f(x)$ is increasing, and $g(x)$ is increasing, then $f(x)g(x)$ is increasing.
   (g) If $f(x)$ is increasing, and $g(x)$ is decreasing, then $f(g(x))$ is decreasing.

3. Find the global maximum and minimum of the given function on the interval provided:
   (a) $f(x) = \sqrt{9 - x^2}$, $[-1, 2]$
   (b) $g(x) = x - 2\cos(x)$, $[-\pi, \pi]$

4. Find the regions where f is increasing/decreasing: $f(x) = \frac{x}{(1+x)^2}$

5. For each function below, determine (i) where f is increasing/decreasing, (ii) where f is concave up/concave down, and (iii) find the local extrema.
   (a) $f(x) = x^3 - 12x + 2$
   (b) $f(x) = x\sqrt{6 - x}$
   (c) $f(x) = x - \sin(x)$, $0 < x < 4\pi$

6. Suppose $f(3) = 2$, $f'(3) = \frac{1}{2}$, and $f'(x) > 0$ and $f''(x) < 0$ for all x.

(a) Sketch a possible graph for f.

(b) How many roots does f have? (Explain):

(c) Is it possible that $f'(2) = 1/3$? Why?

7. Let $f(x) = 2x + e^x$. Show that f has exactly one real root.

8. Suppose that $1 \leq f'(x) \leq 3$ for all $0 \leq x \leq 2$, and $f(0) = 1$. What is the largest and smallest that $f(2)$ can possibly be?

9. Linearize at $x = 0$: $y = \sqrt{x+1}e^{-x^2}$. Use the linearization to estimate $\sqrt{\frac{3}{2}}e^{-\frac{1}{4}}$.

10. Let $f(x) = \sqrt{x} - \frac{x}{3}$ on $[0, 9]$. Verify that the function satisfies all the hypotheses of the Mean Value Theorem, then find the values of c that satisfy its conclusion.

11. Let $f(x) = x^3 - 3x + 2$ on the interval $[-2, 2]$. Verify that the function satisfies all the hypotheses of the Mean Value Theorem, then find the values of c that satisfy its conclusion.

12. Let $f(x) = \tan(x)$. Show that $f(0) = f(\pi)$, but there is no number c in $(0, \pi)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

13. Find the limit, if it exists.

(a) $\displaystyle\lim_{x\to 0} \frac{\sin^{-1}(x)}{x}$

(b) $\displaystyle\lim_{x\to 0} \frac{x3^x}{3^x - 1}$

(c) $\displaystyle\lim_{x\to 0^+} \sin(x)\ln(x)$

(d) $\displaystyle\lim_{x\to 0} \cot(2x)\sin(6x)$

(e) $\displaystyle\lim_{x\to 0^+} x^{\sqrt{x}}$

(f) $\displaystyle\lim_{x\to 0^+} (4x+1)^{\cot(x)}$

14. Verify the given linear approximation (for small x).

(a) $\sqrt[4]{1+2x} \approx 1 + \frac{1}{2}x$

(b) $e^x\cos(x) \approx 1 + x$

15. At 1:00 PM, a truck driver picked up a fare card at the entrance of a tollway. At 2:15 PM, the trucker pulled up to a toll booth 100 miles down the road. After computing the trucker's fare, the toll booth operator summoned a highway patrol officer who issued a speeding ticket to the trucker. (The speed limit on the tollway is 65 MPH).

(a) The trucker claimed that she hadn't been speeding. Is this possible? Explain.

(b) The fine for speeding is \$35.00 plus \$2.00 for each mph by which the speed limit is exceeded. What is the trucker's minimum fine?

16. Let $f(x) = \frac{1}{x}$

(a) What does the Extreme Value Theorem (EVT) say about f on the interval $[0.1, 1]$?

(b) Although f is continuous on $[1, \infty)$, it has no minimum value on this interval. Why doesn't this contradict the EVT?

17. Let f be a function so that $f(0) = 0$ and $\frac{1}{2} \leq f'(x) \leq 1$ for all x. Explain why $f(2)$ cannot be 3 (Hint: You might use a value theorem to help).

18. Optimization practice problems

    (a) Suppose we have two numbers, one is a positive number and the other is its reciprocal. Find the two numbers so that the sum is small as possible.

    (b) Find two positive numbers such that their product is 16 and the sum is as small as possible.

    (c) A 20-inch piece of wire is bent into an L-shape. Where should the bend be made to minimize the distance between the two ends?

    (d) Find the point on the line $y = x$ closest to the point $(1, 0)$.

    (e) A box is constructed out of two different types of metal. The metal for the top and bottom, which are both square, costs \$1.00 per square foot, and the metal for the sides costs \$2.00 per square foot. Find the dimensions that minimize the cost of the box is the box must have a volume of 20 cubic feet.

    (f) A rectangle is to be inscribed between the $x-$axis and the upper part of the graph of $y = 8 - x^2$ (symmetric about the $y-$axis). For example, one such rectangle might have vertices: $(1, 0), (1, 7), (-1, 7), (-1, 0)$ which would have an area of 14. Find the dimensions of the rectangle that will give the largest area.

    (g) What is the smallest possible rea of the triangle that is cut off by the first quadrant and whose hypotenuse is tangent to the curve $y = 4 - x^2$ at some point?

    (h) You're standing with Elvis (the dog) on a straight shoreline, and you throw the stick in the water. Let us label as "A" the point on the shore closest to the stick, and suppose that distance is 7 meters. Suppose that the distance from you to the point A is 10 meters. Suppose that Elvis can run at 3 meters per second, and can swim at 2 meters per second. How far along the shore should Elvis run before going in to swim to the stick, if he wants to minimize the time it takes him to get to the stick?

19. **Graphical Exercises** Please look these problems over as well- They include some graphical analysis.

    - 4.3: 5-6, 7-8, 31-32
    - 4.2: 7