Algebra Solutions

You might recall the following factoring formulas: $x^3 - y^3 = (x-y)(x^2+xy+y^2)$ and $x^3 + y^3 = (x+y)(x^2-xy+y^2)$

1. Factor each polynomial:

   (a) $36z^2 - 81y^4 = 9(2z-3y^2)(2z+3y^2)$

   (b) $16p^2 - 40p + 25 = (4p-5)^2$

   (c) $64 - x^6 = (2^3)^2 - (x^3)^2 = (2^3 - x^3)(2^3 + x^3) = (2-x)(4+2x+x^2)(2+x)(4-2x+x^2)$

   (d) $9(a-4)^2 + 30(a-4) + 25 = (3(a+4)+5)^2 = (3a+17)^2$

   (e) $4x^2y^2 + 28xy + 49 = (2xy+7)^2$

   (f) $(3p^r + 2q^r)^2 - 9 = ((3p^r+2q^r)+3)((3p^r+2q^r)-3)$

   (g) $6p^4 + 7p^2 - 3 = (2p^2+3)(3p^2-1)$

   (h) $2p^2 + 7p - 4 = (p+4)(2p-1)$

2. Find a value of b that will make each expression a perfect square: $k^2 + bk + 16$, $4z^2 + bz + 81$

   For the first one, we would need that $(\frac{b}{2})^2 = 16$. Solving for b, we get that $b = \pm 8$.

   For the second equation, we see that:

$$4z^2 + bz + 81 = 4\left(z^2 + \frac{b}{4}z + \frac{81}{4}\right)$$

   so that $(\frac{b}{8})^2 = \frac{81}{4}$. Solving for b, we get that $b = \pm 36$.

3. Simplify (your answer should not contain negative exponents, where applicable):

   (a)

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}} = \frac{x+1}{x-1}$$

   (b)

$$\frac{1}{h}\left(\frac{1}{(x+h)^2+9} - \frac{1}{x^2+9}\right) = \frac{1}{h}\left(\frac{(x^2+9)-((x+h)^2+9)}{((x+h)^2+9)(x^2+9)}\right) = \frac{-2x-h}{((x+h)^2+9)(x^2+9)}$$

   (c)

$$\left(\frac{y}{y^2-1} - \frac{y}{y^2-2y+1}\right)\left(\frac{y-1}{y+1}\right) = \frac{-2y}{(y+1)^2(y-1)}$$

   (d)

$$\frac{a^{-1}+b^{-1}}{(ab)^{-1}} = \frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{ab}} = b+a$$

   (e)

$$(a+b)^{-1}(a^{-1}+b^{-1}) = \frac{1}{a+b}\cdot\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{1}{a+b}\left(\frac{a+b}{ab}\right) = \frac{1}{ab}$$

   (f)

$$\frac{1}{x^2+x-12} - \frac{1}{x^2-7x+12} + \frac{1}{x^2-16} = \frac{x-5}{(x+4)(x-4)(x-3)}$$

   (g)

$$\frac{2k}{k^2+4k+3} + \frac{3k}{k^2+5k+6} = \frac{k(5k+7)}{(k+3)(x+1)(x+2)}$$

   (h)

$$\frac{\frac{1}{x+y}}{\frac{1}{x}+\frac{1}{y}} = \frac{xy}{(x+y)^2}$$

(i)

$$\frac{5m+25}{10} \cdot \frac{12}{6m+30} = 1$$

(j)

$$\frac{\frac{4a+12}{2a-10}}{\frac{a^2-9}{a^2-a-20}} = \frac{2(a+4)}{a-3}$$

(k)

$$\frac{r^2-r-6}{r^2+r-12} = \frac{r+2}{r+4}$$

(l)

$$\left(3+\frac{2}{y}\right)\left(3-\frac{2}{y}\right) = 9-\frac{4}{y^2}$$

(m)

$$\frac{x^2-y^2}{(x-y)^2} \cdot \frac{x^2-xy+y^2}{x^2-2xy+y^2} \div \frac{x^3+y^3}{(x-y)^4} = x-y$$