Solutions 2.6/2.7/2.8/2.9

1 2.6

4. (a) 2, (b) -2, (c) ∞, (d) $-\infty$, (e) $-\infty$, (f) Vertical: $x = -2, 0, 3$. Horizontal: $y = -2, 2$

8. (Graph. Should have a vertical asymptote at $x = -2$, horizontal asymptotes at $y = -3$ (as $x \to \infty$), and $y = 3$ (as $x \to -\infty$).

12. Divide numerator and denominator by t^3 to get

$$\lim_{t\to\infty} \frac{7 + \frac{4}{t^2}}{2 - \frac{1}{t} + \frac{3}{t^2}} = \frac{7}{2}$$

16. Divide numerator and denominator by t^2 to get:

$$\lim_{t\to\infty} \frac{6 + \frac{5}{t}}{-2 - \frac{5}{t} - \frac{3}{t^2}} = \frac{6}{-2} = -3$$

18. Divide numerator by $-\sqrt{x^2}$ (because $x < 0$), and divide denominator by x to get:

$$\lim_{x\to-\infty} \frac{-\sqrt{1 + \frac{4}{x}}}{4 + \frac{1}{x}} = -\frac{1}{4}$$

20. The quickest way to do this problem is to divide numerator and denominator by $\sqrt{x}$, in which case you would get

$$\lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}} - 1}{\frac{1}{\sqrt{x}} + 1} = -1$$

32. The limit is zero, which we can see by writing

$$\lim_{x\to\infty} \mathrm{e}^{-x^2} = \lim_{x\to\infty} \frac{1}{\mathrm{e}^{x^2}}$$

As $x \to \infty$, so does e^{x^2}.

36. First the vertical asymptotes. There are two of them at $x = \pm 1$, because, as $x \to \pm 1$, $x^2 - 4 \to 3$, and $x^2 - 1 \to 0$. This says that

$$\lim_{x\to\pm 1} \frac{x^2 - 4}{x^2 - 1} = \pm\infty$$

[NOTE: This would NOT be (necessarily) true if the numerator also went to zero!]

For the horizontal asymptotes, divide numerator and denominator by x^2 to get that $y = 1$ is a horizontal asymptote (both for $x \to \infty$ and $x \to -\infty$.

40. For $x \to \infty$, divide the numerator by x and the denominator by $\sqrt{x^2}$ to get:

$$\lim_{x\to\infty} \frac{1 - \frac{9}{x}}{\sqrt{4 + \frac{3}{x} + \frac{2}{x^2}}} = \frac{1}{2}$$

For $x \to -\infty$, divide the numerator by x and the denominator by $-\sqrt{x^2}$ to get:

$$\lim_{x\to-\infty} \frac{1 - \frac{9}{x}}{-\sqrt{4 + \frac{3}{x} + \frac{2}{x^2}}} = -\frac{1}{2}$$

49. The general setting is nice to know (check your book), but I won't ask this on an exam.

52. After t minutes, the concentration of salt in the tank is (put pounds of salt in the numerator, liters of liquid in the denominator):

$$C(t) = \frac{30\frac{\text{grams}}{\text{liter}} \cdot 25t \text{ liters}}{5000 + 25t \text{ liters}} = \frac{30t}{200 + t}\frac{\text{grams}}{\text{liter}}$$

so the limiting concentration is 30 grams per liter (divide numerator and denominator by t).

2 2.7

2. Average velocity is $\dfrac{f(a+h) - f(a)}{h}$ and instantaneous velocity (or simply velocity) is $\displaystyle\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

6. (a) Using the two (equivalent) definitions, we get:

$$m = \lim_{h\to 0} \frac{(-1+h)^3 - (-1)^3}{h} = 3$$

(multiply everything out, and cancel an h).

$$m = \lim_{x\to -1} \frac{x^3 - (-1)^3}{x - 1} = 3$$

(Use the factoring formula for $x^3 + 1^3 = (x + 1)(x^2 - x + 1)$)

(b) The tangent line has the equation: $y - (-1) = 3(x - (-1))$

12. We compute by using our regular definition, $\displaystyle\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$:

$$\lim_{h\to 0} \frac{1 + (a+h) + (a+h)^2 - 1 - a - a^2}{h} =$$

$$\lim_{h\to 0} \frac{1 + a + h + a^2 + 2ah + h^2 - 1 - a - a^2}{h} =$$

$$\lim_{h\to 0}\frac{h+2ah+h^2}{h}=\lim_{h\to 0}1+2a+h=1+2a$$

For the second part, we only need to evaluate our expression $(1+2a)$ for the given values of a: At $a=-1$, the velocity is -1, at $a=\frac{-1}{2}$, we get 0 and at $a=1$, we get 3.

15. Initially, the velocity was 0. The car was going faster at C because the slope of the tangent line there is steeper than at B. Near A, the tangent lines are becoming steeper, so the car was speeding up. Near B, the tangent lines are becoming less steep, so the car was slowing down. The steepest tangent near C is at C, so that is where the car had finished speeding up, and was about to start slowing down. Between D and E, the slope was zero, so the car did not move.

20. This is almost identical to Problem 6, section 2.1. We are computing average velocities, then we'll compute the instantaneous velocity at $t=4$.
    (a) (i) -1, (ii) -0.5, (iii) 1, (iv) 0.5
    (b) The instantaneous velocity is at $t=4$ is 0.

3 2.8

2. As $h\to 0$, the slope of PQ is getting steeper, so

$$0<\frac{f(4)-f(2)}{2}<f(3)-f(2)<f'(2)$$

6. Your graph should go through the origin, have a hill with the top at $x=1$, dip down slightly, then increase again.

10(a). Be sure to get a common denominator, then simplify:

$$\lim_{h\to 0}\frac{\frac{a+h}{1+2(a+h)}-\frac{a}{1+2a}}{h}=$$

$$\lim_{h\to 0}\frac{\frac{(a+h)(1+2a)-a(1+2a+2h)}{(1+2a+2h)(1+2a)}}{h}=$$

$$\lim_{h\to 0}\frac{1}{(1+2a+2h)(1+2a)}=\frac{1}{(1+2a)^2}$$

16. Similar to the last problem. Get a common denominator and simplify, simplify, simplify!

$$\lim_{h\to 0}\frac{\frac{a+h}{2(a+h)-1}-\frac{a}{2a-1}}{h}=$$

$$\lim_{h\to 0}\frac{\frac{(a+h)(2a-1)-a(2a+2h-1)}{(2a+2h-1)(2a-1)}}{h}=$$

$$\lim_{h\to 0}\frac{1}{(2a+2h-1)(2a-1)}=\frac{1}{(2a-1)^2}$$

20. $f(x)=x^3, a=2$

22. $f(x)=\cos(x), a=3\pi$

24. $f(x)=3^x, a=0$

4 2.9

4. (a)=II, (b)=IV, (c)=I, (d)=III

6. Your graph should be increasing, going through the origin (e.g., a line)

8. Your graph should have a horizontal asymptote where f has a cusp (or point). To the left of the asymptote, your graph is positive and increasing. To the right, the graph is initially negative, goes through 0 where f has a dip, then increases after that.

12. The graph is broken line segments (with open circles at the endpoints). (We'll do this in class).

22. (This one was done in class)

$$\lim_{h\to 0}\frac{x+h+\sqrt{x+h}-x-\sqrt{x}}{h}=$$

$$\lim_{h\to 0}\frac{h+\sqrt{x+h}-\sqrt{x}}{h}=$$

$$1+\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=1+\frac{1}{2\sqrt{x}}$$

The domain of f is $x\geq 0$. The domain of $f'(x)$ is $x>0$

24. Get a common denominator and simplify!

$$\lim_{h\to 0}\frac{\frac{x+h+1}{x+h-1}-\frac{x+1}{x-1}}{h}=$$

$$\lim_{h\to 0}\frac{\frac{(x+h+1)(x-1)-(x+1)(x+h-1)}{(x+h-1)(x-1)}}{h}=$$

$$\frac{-2}{(x-1)^2}$$

so the domain of f is the domain of f', $x\neq 1$.

32. This is an estimation problem. If you want the solution, stop by and see me, but I won't ask this type of problem on the exam.