Summary: 4.1/4.3: Extreme Values

1. Know the definitions of global max/global min, local max/local min.

2. Definition: The critical values of f are the numbers x where $f'(x) = 0$ or where $f'(x)$ is not defined.

3. The Extreme Value Theorem. If f is continuous on $[a, b]$, then f attains a maximum and a minimum on $[a, b]$.

   This is an Existence Theorem- It tells us a max/min exists, but not where to find them.

4. Fermat's Theorem: If f has a local max/min at $x = a$, and f is differentiable at $x = a$, then $f'(a) = 0$.

   Fermat's Theorem gives us the justification for the procedure that follows.

5. Procedure for finding a global max/min on $[a, b]$ if f is continuous: Find the candidates, and check the corresponding $y-$values.

   - Compute $f(a)$ and $f(b)$.
   - Find the critical values of f and compute the corresponding $y-$values.

   The largest of these is the global max, the smallest is the global min.

   Note: If the problem does not satisfy the Extreme Value Theorem, we take each function on a case-by-case basis. In these cases, the signs of the first and second derivative (use a table!) can be very revealing.

6. Procedure for locating local max/min. We have a candidate for the local max/min, $x = a$ ($f'(a) = 0$ or does not exist).

   (a) The First Derivative Test.
   - If f' changes sign from $+$ to $-$, $f(a)$ is a local max.
   - If f' changes sign from $-$ to $+$, $f(a)$ is a local min.
   - If f' does not change sign, $f(a)$ is neither.

   (b) The Second Derivative Test.
   - If $f''(a) > 0$ (concave up), $f(a)$ is a local min.
   - If $f''(a) < 0$ (concave down), $f(a)$ is a local max.
   - If $f''(a) = 0$ or does not exist, this test fails. (We can't say anything about $x = a$- Try another test.)

7. Recall that the first and second derivatives tell us the following about the graph of a function:

   - If $f'(a) > 0$, f is increasing at $x = a$.
   - If $f'(a) < 0$, f is decreasing at $x = a$.
   - If $f'(a) = 0$, f is "locally constant" at $x = a$.
   - If $f''(a) > 0$, f is concave up at $x = a$.
   - If $f''(a) < 0$, f is concave down at $x = a$.

8. You might also review the method (using a table) of finding where a polynomial is positive or negative.