Summary: 4.2, The Mean Value Theorem

1. Rolle's Theorem: Let f be a function that satisfies the following three properties:
   - f is continuous on $[a, b]$.
   - f is differentiable on (a, b).
   - $f(a) = f(b)$.

   Then there is a c in (a, b) so that $f'(c) = 0$.

   Remarks:
   - Once you're familiar with this section, think of Rolle's Theorem as a special case of the Mean Value Theorem.
   - Rolle's Theorem is an existence theorem- It does not say how to find c, only that a c exists.
   - Rolle's Theorem is introduced as a method for proving the main theorem of this section: The Mean Value Theorem.

2. The Mean Value Theorem. Let f be a function that satisfies the following two properties:
   - f is continuous on $[a, b]$.
   - f is differentiable on (a, b).

   Then there is a c in (a, b) such that
   $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

   You can remember this theorem by thinking that the quantity $\frac{f(b)-f(a)}{b-a}$ is a kind of average- Recall that in physics, if f is position, then the fraction represented average velocity.

3. Applications of the Mean Value Theorem:
   - If $f'(x) = 0$ for all x in (a, b), then f is constant (e.g., $f(x) = k$, for some k) on (a, b).
   - If $f'(x) = g'(x)$, then $f(x) = g(x) + c$. That is, if two derivatives are the same, there corresponding functions are constant multiples of each other.