

## Summary of 4.4: L'Hospital's Rule

L'Hospital's Rule is a very popular method for computing limits. In this section, we learn the rule and some algebraic techniques for translating problems into the right form for L'Hospital's rule.

1. Definition: A limit is of an indeterminate form if:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

Remarks on the definition:

- $x \rightarrow a$  can be replaced by  $x \rightarrow \pm\infty$ .
  - $\frac{\infty}{\infty}$  can be replaced by  $\frac{\pm\infty}{\pm\infty}$
2. L'Hospital's Rule: Suppose  $f$  and  $g$  are differentiable and  $g(x) \neq 0$  near  $a$  (except possibly at  $x = a$ ). Then if the limit is an indeterminate form, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

provided that this limit exists.

3. Conversion Types:

- If the limit is  $0 \cdot \infty$ , we can convert it to a limit of the form  $\frac{0}{0}$ .
- If the limit is  $\infty - \infty$ , we can convert it to either  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ .
- If the limit is  $0^0$ ,  $\infty^0$  or  $\infty^\infty$ , by taking logs, we can convert to  $0 \cdot \infty$ , and continue from there.