Sample Exam Questions

Disclaimer: These questions are intended to give you a variety of typical-to-harder type questions, and to look at questions "context-free".

1. Find the local maximums and minimums: $f(x) = x^3 - 3x + 1$ Show your answer is correct by using both the first derivative test and the second derivative test.

2. Find dy in terms of x and dx: $y = \frac{x}{\sin(2x)}$.

3. Compute the derivative of y with respect to x:

   (a) $y = \sqrt[3]{2x+1}\sqrt[5]{3x-2}$

   (b) $y = \frac{1}{1+u^2}$, where $u = \frac{1}{1+x^2}$

   (c) $\sqrt[3]{y} + \sqrt[3]{x} = 4xy$

   (d) $\sqrt{x+y} = \sqrt[3]{x-y}$

   (e) $y = \sin(2\cos(3x))$

   (f) $y = (\cos(x))^{2x}$

   (g) $y = (\tan^{-1}(x))^{-1}$

   (h) $y = \sin^{-1}(\cos^{-1}(x))$

   (i) $y = \log_{10}(x^2 - x)$

   (j) $y = x^{x^2+2}$

4. List the three items we need to check to see if a function $f(x)$ is continuous at $x = a$.

5. Derive the formula for the derivative of $y = \sec^{-1}(x)$.

6. Write the equation of the line tangent to $x = \sin(2y)$ at $x = 1$.

7. If a hemispherical bowl with radius 1 foot is filled with water to a depth of x inches, then the volume of the water in the bowl is given by:

$$V = \frac{\pi}{3}\left(36x^2 - x^3\right) \quad \text{cubic inches}$$

   If the water flows out a hole in the bottom at the rate of 36π cubic inches per second, how fast is the water level decreasing when $x = 6$ inches?

8. Compute the limit, if it exists. You may use any method (except a numerical table).

   (a) $\displaystyle\lim_{x\to 0} \frac{x - \sin(x)}{x^3}$

   (b) $\displaystyle\lim_{x\to 4^+} \frac{x-4}{|x-4|}$

   (c) $\displaystyle\lim_{x\to -\infty} \sqrt{\frac{2x^2-1}{x+8x^2}}$

   (d) $\displaystyle\lim_{x\to\infty} \sqrt{x^2+x+1} - \sqrt{x^2-x}$

   (e) $\displaystyle\lim_{h\to 0} \frac{(1+h)^{-2} - 1}{h}$

   (f) $\displaystyle\lim_{x\to 1} \frac{x^{1000} - 1}{x - 1}$

   (g) $\displaystyle\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)}$

   (h) $\displaystyle\lim_{x\to 1} x^{\frac{1}{1-x}}$

9. Determine all vertical/horizontal asymptotes and critical points of $f(x) = \frac{2x^2}{x^2-x-2}$

10. Why does Newton's Method fail, if:

    (a) $f(x) = x^2 + 1$

    (b)

$$f(x) = \begin{cases} \sqrt{x}, & \text{if } x \geq 0 \\ -\sqrt{-x}, & \text{if } x < 0 \end{cases}$$

11. Find values of m and b so that (1) f is continuous, and (2) f is differentiable.

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

12. Find the local and global extreme values of $f(x) = \frac{x}{x^2+x+1}$ on the interval $[-2, 0]$.

13. Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$

14. Suppose f is differentiable so that:

$$f(1) = 1,\ f(2) = 2,\ f'(1) = 1\ f'(2) = 2$$

    If $g(x) = f(x^3 + f(x^2))$, evaluate $g'(0)$.

15. Find y'' by implicit differentiation:

$$x^2 + 6xy + y^2 = 8$$

16. Let

$$x^2y + a^2xy + \lambda y^2 = 0$$

    (a) Let a and λ be constants, and let y be a function of x. Calculate $\frac{dy}{dx}$:

    (b) Let x and y be constants, and let a be a function of λ. Calculate $\frac{da}{d\lambda}$:

17. Show that $x^4 + 4x + c = 0$ has at most one solution in the interval $[-1, 1]$.

18. True or False, and give a short explanation.

(a) If $f'(r)$ exists, then

$$\lim_{x\to r} f(x) = f(r)$$

(b) If f and g are differentiable, then:

$$\frac{d}{\mathrm{dx}}(f(g(x)) = f'(x)g'(x)$$

(c) If $f(x) = x^2$, then the equation of the tangent line at $x = 3$ is:

$$y - 9 = 2x(x - 3)$$

(d)

$$\lim_{\theta\to\frac{\pi}{3}} \frac{\cos(\theta) - \frac{1}{2}}{\theta - \frac{\pi}{3}} = -\sin\left(\frac{\pi}{3}\right)$$

(e) There is no solution to $\mathrm{e}^x = 0$

(f) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

(g) $5^{\log_5(2x)} = 2x$, for $x > 0$.

(h) $\frac{d}{dx}\ln(|x|) = \frac{1}{x}$, for all $x \neq 0$.

(i) $\frac{d}{dx}10^x = x10^{x-1}$

(j) If $x > 0$, then $(\ln(x))^6 = 6\ln(x)$

19. Find the domain of $\ln(x - x^2)$:

20. Find the value of c guaranteed by the Mean Value Theorem, if $f(x) = \frac{x}{x+2}$ on the interval $[1, 4]$.

21. Compute the limit, without using L'Hospital's Rule. $\lim_{x\to 7} \dfrac{\sqrt{x+2} - 3}{x - 7}$

22. For what value(s) of c does $f(x) = cx^4 - 2x^2 + 1$ have both a local maximum and a local minimum?

23. If $f(x) = \sqrt{1 - 2x}$, determine $f'(x)$ *by using the definition of the derivative.*

24. Use Newton's Method to find the absolute minimum of $f(x) = x^6 + 2x^2 - 8x + 3$ correct to four decimal places.

25. A *point of inflection* for a function f is the x value for which $f''(x)$ changes sign (either from positive to negative or vice versa).

    (a) If f'' is continuous, then $f''(x) = 0$ at an inflection point. What theorem did we have that proves this?

    (b) Find constants a and b so that $(1, 6)$ is an inflection point for $y = x^3 + ax^2 + bx + 1$.

26. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f(3) = 2$ and $f'(6) = 7$. Find $F'(3)$.

27. Let $G(x) = h(\sqrt{x})$. Then where is G differentiable? Find $G'(x)$.

28. If position is given by: $f(t) = t^4 - 2t^3 + 2$, find the times when the acceleration is zero. Then compute the velocity at these times.

29. If $y = \sqrt{5t - 1}$, compute y'''.

30. Find a second degree polynomial so that $P(2) = 5$, $P'(2) = 3$, and $P''(2) = 2$.

31. If $f(x) = (2 - 3x)^{-1/2}$, find $f(0)$, $f'(0)$, $f''(0)$.

32. Car A is traveling west at 50 mi/h, and car B is traveling north at 60 mi/h. Both are headed for the intersection between the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

33. Compute Δy and dy for the value of x and Δx: $f(x) = 6 - x^2$, $x = -2$, $\Delta x = 0.4$.

34. Find the linearization of $f(x) = \sqrt{1 - x}$ at $x = 0$.

35. Find $f'(x)$ directly from the definition of the derivative (using limits):

    (a) $f(x) = \sqrt{3 - 5x}$

    (b) $f(x) = x^2$

36. True/False: The equation of the tangent line to $f(x) = e^x - e^{-2x}$ at $x = 0$ is:

$$y - 0 = (e^x + 2e^{-2x})(x - 0)$$

37. Differentiate:

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ -\sqrt{x} & \text{if } x < 0 \end{cases}$$

Is f differentiable at $x = 0$? Explain.

38. $f(x) = |\ln(x)|$. Find $f'(x)$.

39. $f(x) = x\mathrm{e}^{g(\sqrt{x})}$. Find $f'(x)$.

40. Let $f(3) = 2$, and $f'(3) = -1$. If g is the inverse of f, (a) At what ordered pair can we evaluate the derivative of g? (b) Compute that derivative.

41. Find a formula for dy/dx: $x^2 + xy + y^3 = 0$.

42. Show that 5 is a critical number of $g(x) = 2+(x-5)^3$, but that g does not have a local extremum there.

43. Find the slope of the tangent line to the following at the point (3,4): $x^2 + \sqrt{y}x + y^2 = 19$

44. Find the critical values: $f(x) = |x^2 - x|$

45. Does there exist a function f so that $f(0) = -1$, $f(2) = 4$, and $f'(x) \le 2$ for all x?

46. Milk is being produced in a spherical cow so that it's volume increases at a rate of 100 cm^3/s. How fast is the radius of the cow increasing when its diameter is 50 dm?

47. Linearize $f(x) = \sqrt{1+x}$ at $x = 0$.

48. Find dy is $y = \sqrt{1-x}$ and evaluate dy if $x = 0$ and $dx = 0.02$.

49. Fill in the question marks: If f'' is positive on an interval, then f' is ? and f is ?.

50. If $f(x) = x - \cos(x)$, x is in $[0, 2\pi]$, then find the value(s) of x for which
    (a) $f(x)$ is greatest and least.
    (b) $f(x)$ is increasing most rapidly.
    (c) The slopes of the lines tangent to the graph of f are increasing most rapidly.

51. Show there is *exactly* one root to: $\ln(x) = 3 - x$ between 2 and e, then use Newton's Method to approximate it (accurate to 3 decimal places).

52. Use differentials to find a formula for the approximate volume of a thin cylindrical can with height h, inner radius r, and thickness Δr.

53. Sketch the graph of a function that satisfies all of the given conditions:
    (a) $f(1) = 5, f(4) = 2$
    (b) $f'(1) = f'(4) = 0$,
    (c) $\lim_{x\to 2^+} f(x) = \infty$
    (d) $\lim_{x\to 2^-} f(x) = 3$, and $f(2) = 4$

54. Find the domain of $f(x) = \log_3(x^4 - 8x^3 + 15x^2)$

55. Find the dimensions of the rectangle of largest area that has its base on the x-axis and the other two vertices above the x-axis on the parabola $y = 8 - x^2$.

56. Use Newton's Method to approximate x_1, x_2, x_3, for $x_0 = 0.5$ and $f(x) = 2x - \cos(x)$.