Review Problems: Chapter 11

1. What does it mean to say that a series "converges" (I'm looking for the definition; be sure you define any notation you use).

2. Does the given sequence or series converge or diverge? (In this case, you do not have to specify absolute or conditional convergence).

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$

   (b) $\left\{\ln\left(\frac{n+2}{n}\right)\right\}$

   (c) $\left\{\frac{n}{1+\sqrt{n}}\right\}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+1}{n^3-1}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{5-2\sqrt{n}}{n^3}$

   (f) $\displaystyle\sum_{n=1}^{\infty} (-6)^{n-1}5^{1-n}$

   (g) $\left\{\frac{n!}{(n+2)!}\right\}$

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdots(2n-1)}{5^n n!}$

   (i) $\displaystyle\sum_{n=2}^{\infty} \frac{3^n+2^n}{6^n}$

   (j) $\left\{\sin\left(\frac{n\pi}{2}\right)\right\}$

   (k) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

   (l) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n\sqrt{n}}$

   (m) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^{2n}}{n^2 9^n}$

   (n) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)2^n}$

3. Find the sum of the series. Hint on the last one- It is a telescoping series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{2^{2n}}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(x-3)^{2n}}{3^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4}{n+4} - \frac{4}{n+5}\right)$

4. Find the radius of convergence. For the last two, include the interval of convergence.

   (a) $\displaystyle\sum \frac{n!x^n}{1\cdot 3\cdot 5\cdots(2n-1)}$

   (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n^2 5^n}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}$

5. Find a series for the following (centered at 0).

   (a) $\dfrac{1}{1-3x}$

   (b) $\dfrac{x^2}{1+x}$

6. True or False, and give a short reason:

   (a) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum a_n$ is convergent.

   (b) If $\sum c_n 6^n$ is convergent, so is $\sum c_n(-2)^n$.

   (c) The Ratio Test can be used to determine if a $p-$series is convergent.

   (d) If $0 \le a_n \le b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

   (e) $0.9999999\ldots = 1$

   (f) If $a_n > 0$ and $\sum a_n$ converges, then $\sum(-1)^n a_n$ converges.

7. Suppose that $\sum_{n=0}^{\infty} c_n(x-1)^n$ converges when $x=3$ and diverges when $x=-2$. What can be said about the convergence or divergence of the following?

(a) $\sum c_n$ (b) $\sum (-1)^n c_n$ (c) $\sum c_n 3^n$

8. Let $a_n = \dfrac{2n}{3n+1}$

   (a) Determine whether $\{a_n\}$ is convergent.

   (b) Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.

9. Same as the previous problem, but use $a_n = \dfrac{1+2^n}{3^n}$

10. What is a formula for the sum: $1 + r + r^2 + \cdots + r^{99}$? (Hint: Let S be equal to the quantity. What happens when you subtract rS?)

11. Explain the difference between absolute and conditional convergence. Which is "better" and why?

12. Determine whether each series converges or diverges. In this question, also determine if the series converges absolutely or conditionally.

    (a) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{\pi}{2} n\right)$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

    (c) $\displaystyle\sum_{k=1}^{\infty} \frac{\cos(k)}{k^3}$

    (d) $\displaystyle\frac{1}{2} - \frac{4}{2^3+1} + \frac{9}{3^3+1} - \frac{16}{4^3+1} + \cdots$